E. DEWEY.

The Association
of
Engineering and Shipbuilding Draughtsmen.

EPICYCLIC GEARING.

By R. McCRAE, B.Sc. (Eng.), A.M.I.Mech.E.

Published by The Draughtsman Publishing Co., Ltd.,
96 St. George's Square, London, S.W. 1.

SESSION 1937-38.

Printed by Milne, Tannahill & Methven, Ltd. (T.U.), 12-14 Mill Street, Perth.

ADVICE TO INTENDING AUTHORS OF A.E.S.D. PRINTED PAMPHLETS

Pamphlets submitted to the National Technical Sub-Committee for consideration with a view to publication in this series should not exceed 9,000 to 10,000 words and about 20 illustrations, making a pamphlet of about 32 to 40 pages.

Drawings for illustrations should be done either on a good plain white paper or tracing cloth, deep black Indian ink being used. For ordinary purposes they should be made about one-and-a-half times the intended finished size, and it should be arranged that wherever possible these shall not be greater than a single full page of the pamphlet, as folded pages are objectionable, although, upon occasions, unavoidable. Where drawings are made larger, involving a greater reduction, the lines should be made slightly heavier and the printing rather larger than normal as the greater reduction tends to make the lines appear faint and the printing excessively small in the reproduction. In the case of charts or curves set out on squared paper, either all the squares should be inked in or the chart or curve should be retraced and the requisite squares inked in.

Authors of pamphlets are requested to adhere to the standard symbols of the British Standards Institution, where lists of such standard symbols have been issued, as in the case of the electrical and other industries, and also to the *British Standard Engineering Symbols and Abbreviations*, published by the B.S.I. in 1934 at 3/6. Attention might also be given to mathematical notation, where alternative methods exist, to ensure the minimum trouble in setting up by the printer.

Manuscripts, in the first instance, should be submitted to the Editor, "The Draughtsman," 96 St. George's Square, Westminster, London, S.W.1.

For Pamphlets a grant of £10 is made to the Author, but special consideration is given in the case of much larger pamphlets which may involve more than the usual amount of preparation.

The Draughtsman Publishing Co., Ltd., accepts no responsibility for the formulae or opinions expressed in its Technical Publications.

The Association
of
Engineering and Shipbuilding
Draughtsmen.

EPICYCLIC GEARING.

By R. McCRAE, B.Sc. (Eng.), A.M.I.Mech.E.

Published by The Draughtsman Publishing Co., Ltd.,
96 St. George's Square, London, S.W.1.

SESSION 1937-38.

Printed by Milne, Tannahill & Methven, Ltd. (T.U.), 12-14 Mill Street, Perth.

EPICYCLIC GEARING.

By R. McCRAE, B.Sc. (Eng.), A.M.I.Mech.E.

INTRODUCTORY.

THE main advantage possessed by epicyclic gearing is that it provides a means of obtaining very high reduction ratios in a very compact form and without the necessity of using small diameter pinions with large diameter spur wheels. The field of application of this type of gearing has shown a distinct tendency to expand in recent years. One of the main factors tending to hinder this expansion has been the feeling of awe and fear which the mere mention of the word EPICYCLIC seems to conjure up in the minds of many engineers, and it is hoped that this pamphlet, by explaining the fundamental principles involved, will dispel this fear for good and perhaps pave the way for a further extension of its uses to all types of engineering work involving the use of toothed gearing.

Velocity Ratios.

In *ordinary gears* all wheels rotate about *stationary centres.* In *epicyclic gears* some of the wheels *move bodily in addition to rotating about their own axes.* It is therefore necessary to consider an additional member known as the *arm* which corresponds to the *fixed frame* in an ordinary train of gears.

Consider the simple case of two wheels A and B gearing externally, their centres being connected by the arm P. In Fig. 1, let *a* and *b* be points on A and B which are originally in contact. Keeping the arm P fixed, turn the wheel A through a clockwise angle θ. Then the wheel B will turn through a counter clockwise angle ϕ such that $\phi = \frac{r_A}{r_B}\theta$ where r_A and r_B are the pitch circle radii of wheels A and B respectively. This position is shown in Fig. 2. Now suppose the whole gear to be locked "solid" and to rotate bodily about the axis of wheel A until the point *a* comes back to its original position, as shown in Fig. 3. It will be observed that the arm P has turned through a counter-clockwise (or negative) angle θ and that the wheel B has turned through a counter clockwise (or negative) angle $(\theta + \phi)$. (The angle turned through by the wheel B will be made more clear by considering the movement to be made in two stages. First of all, let the axis of the wheel B turn through

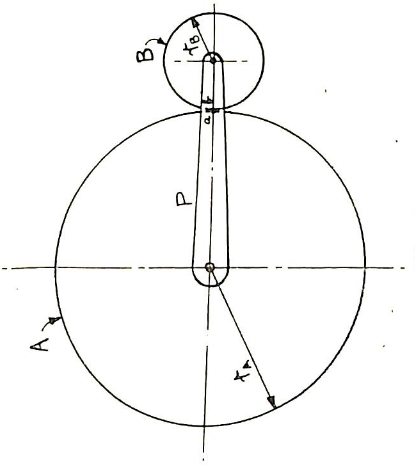


Fig. 1.

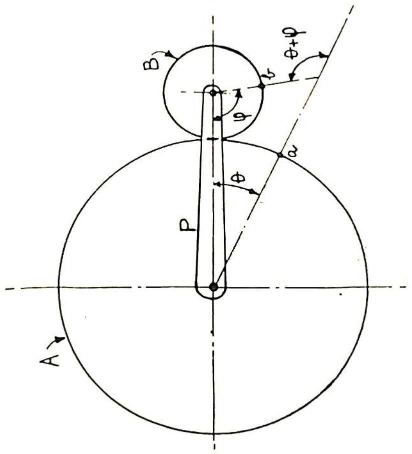


Fig. 2.

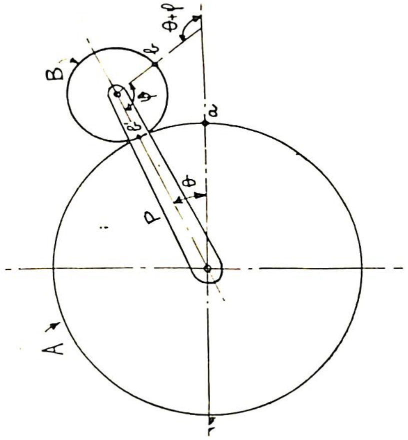


Fig. 3.

the negative angle θ relative to the axis of the wheel A. This would bring the point b to b^1. Then let the wheel B turn through the negative angle ϕ relative to its own axis, bringing the point b^1 to b and making the total negative angle turned through by the wheel B equal to $\theta + \phi$).

FIXED ↓	TURNS OF			REMARKS.
	A	B	P	
P	$+\theta$	$-\phi$	0	Fig. 2.
A	0	$-\phi-\theta$	$-\theta$	Fig. 3.
B	$\theta+\phi$	0	ϕ	Similarly.

The table gives the foregoing results arranged in tabular form. It will be observed that the values in the second line of the table can be obtained from those in the first line merely by subtracting $+\theta$ from each item. Similarly, to find the conditions when B is fixed, subtract $-\phi$ from each item in the first line. Hence when dealing with an epicyclic train of gears it is sufficient to fix the arm, treat the gear as an ordinary train, tabulate the rotations of the various wheels and obtain the rotations for the epicyclic train by simple subtraction. Observation of the following simple set of rules reduces the calculation of velocity ratios for epicyclic trains of gears to a merely mechanical process.

1. Sketch the gear and assign a letter to each wheel *and to the arm.*
2. Consider the arm fixed and give *any convenient wheel* one positive revolution.
3. Tabulate the corresponding revolutions of each of the other wheels, *treating the gear as an ordinary fixed centre train.*
4. Subtract the revolutions of the fixed wheel from each item of the table. Then velocity ratio of gear

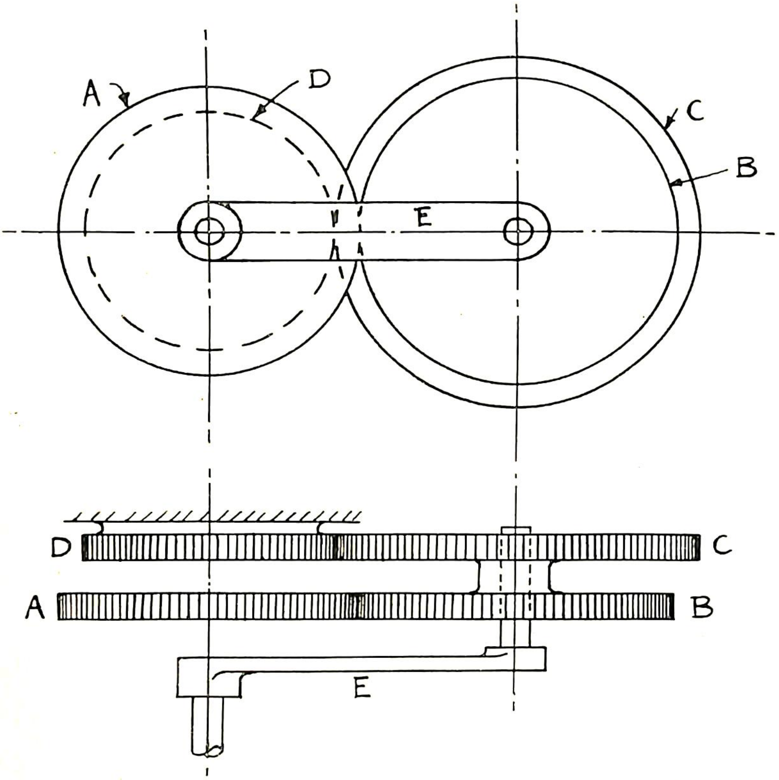
$$= \frac{\text{Revs. of follower}}{\text{Revs. of driver}}$$

The following set of examples illustrates the application of these rules :—

Example 1—Simple Reverted Train.

Find the velocity of the train shown in Fig. 4.

N.B.—The word "reverted" simply means that the axis of the first wheel coincides with the axis of the last wheel, literally "coming back to the same centre."



A—50 Teeth	(Driver)
B—51 Teeth	Compound
C—50 Teeth	Planet Pinion
D—49 Teeth	(Fixed Wheel)

Fig. 4.

Considering the arm E fixed give (B, C) one positive revolution. Then A will make $-{}^{51}/_{50}$ revs. and D will make $-{}^{50}/_{49}$ revs. These results may be tabulated thus :—

FIXED. ↓	REVS. OF			
	A	B, C	D	E
E	$-{}^{51}/_{50}$	$+1$	$-{}^{50}/_{49}$	0
D	$-{}^{51}/_{50}+{}^{50}/_{49}$	$1+{}^{50}/_{49}$	0	${}^{50}/_{49}$
	Driver		Fixed Wheel	Follower

To obtain the second line of the table, subtract the revolutions of the fixed wheel D, i.e., $-{}^{50}/_{49}$ from each item in the first line.

$$\text{Then velocity ratio of gear} = \frac{\text{Revs. of follower E}}{\text{Revs. of driver A}}$$

$$= \frac{{}^{50}/_{49}}{-{}^{51}/_{50} + {}^{50}/_{49}}$$

$$= \frac{1}{(-{}^{51}/_{50} \times {}^{49}/_{50}) + 1}$$

$$= \frac{1}{-{}^{2499}/_{2500} + 1}$$

$$= \frac{1}{{}^{1}/_{2500}}$$

$$= \frac{2500}{1}$$

This is a very compact arrangement for high reduction ratios. Generally for a ratio of n to 1 (if n is a perfect square) the numbers of the teeth will be $(\sqrt{n}-1)$, $\sqrt{n}$, $\sqrt{n}$ and $(\sqrt{n}+1)$ in wheels D, C, A and B respectively. By comparison with example 1, it will be seen that the value of the train will then be

$$\frac{\text{Revs. of follower E}}{\text{Revs. of driver A}} = \frac{\dfrac{\sqrt{n}}{\sqrt{n}-1}}{-\dfrac{\sqrt{n}+1}{\sqrt{n}} + \dfrac{\sqrt{n}}{\sqrt{n}-1}}$$

$$= \frac{1}{-\left(\dfrac{\sqrt{n}+1}{\sqrt{n}} \times \dfrac{\sqrt{n}-1}{\sqrt{n}}\right) + 1}$$

$$= \frac{1}{-\left(\dfrac{n-1}{n}\right) + 1}$$

$$= \frac{1}{\dfrac{-n+1+n}{n}}$$

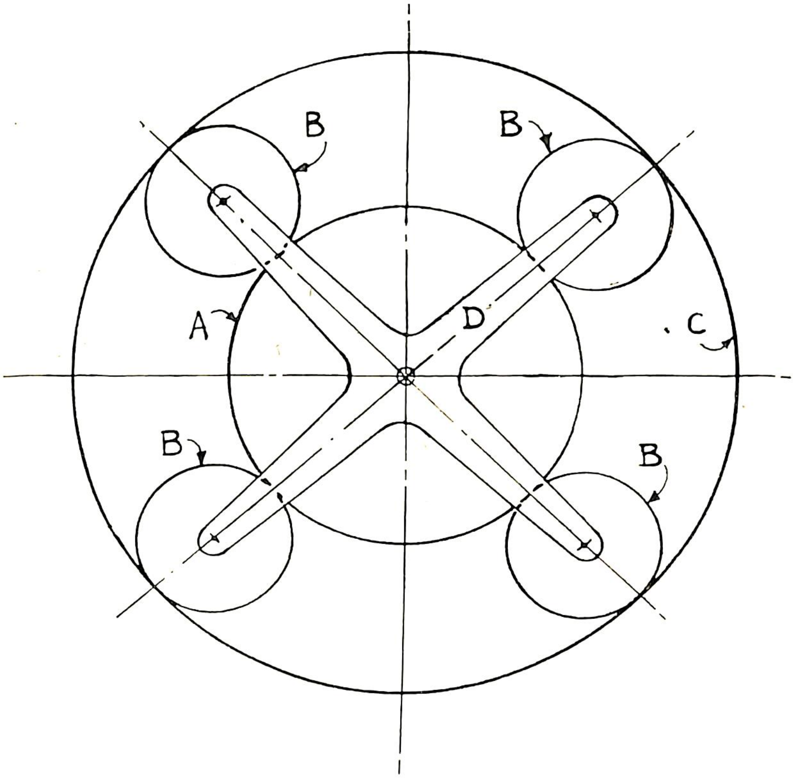
$$= \frac{1}{\dfrac{1}{n}}$$

$$= \frac{n}{1}.$$

A case was recently brought to the notice of the writer where an epicyclic gear of this type was used in conjunction with a worm reduction gear to give an overall ratio of 4000 to 1. The arrangement was so compact that externally the appearance of the casing did not differ from that of a normal worm drive.

Example 2.—Bicycle Gear. Fig. 5.

This is a type of epicyclic gear with which many draughtsmen, who, of choice or necessity, prefer the "push bike" in this age of speed, will be familiar.



A—Wheel with 14 Teeth fixed to Fork of Cycle.

B—4 Planet Pinions, each with 12 Teeth, running freely on Pins from Arm D, which is fixed to the Axle.

C—Annular Wheel with 38 Teeth fixed to Hub.

Fig. 5.

Fixed ↓	Revs. of			
	A	B	C	D
D	$-{}^{12}/_{14}$	$+1$	$+{}^{12}/_{38}$	0
A	0	$1+{}^{12}/_{14}$	${}^{12}/_{38}+{}^{12}/_{14}$	${}^{12}/_{14}$
	Fixed Wheel		Follower	Driver

As before, the first line of the table is obtained by fixing the arm D and giving the pinion B one positive revolution. To obtain the second line of the table, subtract the revolutions of the fixed wheel, i.e., $-{}^{12}/_{11}$ from each item in the first line.

$$\text{Then velocity ratio of gear} = \frac{\text{Revs. of follower C}}{\text{Revs. of driver D}}$$

$$= \frac{{}^{12}/_{38} + {}^{12}/_{14}}{{}^{12}/_{14}}$$

$$= \frac{1{\cdot}37}{1}$$

Suppose now we require the speed of the axle when cycling at 10 miles per hour with a 28″ diam. bicycle wheel.

$$\text{Circumference of wheel} = \frac{\pi \times 28}{12} \text{ feet.}$$

$$= 7{\cdot}32 \text{ feet.}$$

$$\text{Speed of hub} = \frac{52{,}800}{60 \times 7{\cdot}32} \text{ revs. per min.}$$

$$\therefore \quad \text{Speed of axle} = \frac{52{,}800}{60 \times 7{\cdot}32 \times 1{\cdot}37} \text{ revs. per min.}$$

$$= \mathbf{87{\cdot}7} \text{ revs. per min.}$$

Example 3.—Ford Car Gear. Fig. 6.

Consider next the ingenious epicyclic gear known as the Ford car gear, giving two forward speeds and a reverse speed.

The arrangement of the gear is shown diagrammatically in Fig. 6. The flywheel A acts as the arm of the epicyclic gear and carries three pins, two of which are indicated. Each pin carries a compound planet pinion BDF; for the sake of clearness, only one of these pinions is shown. B gears with a wheel C which is connected through a clutch (not shown) with the driven shaft. The driven shaft in turn is connected with the rear axle through the bevel wheels H and J. The differential gear between the driven shaft

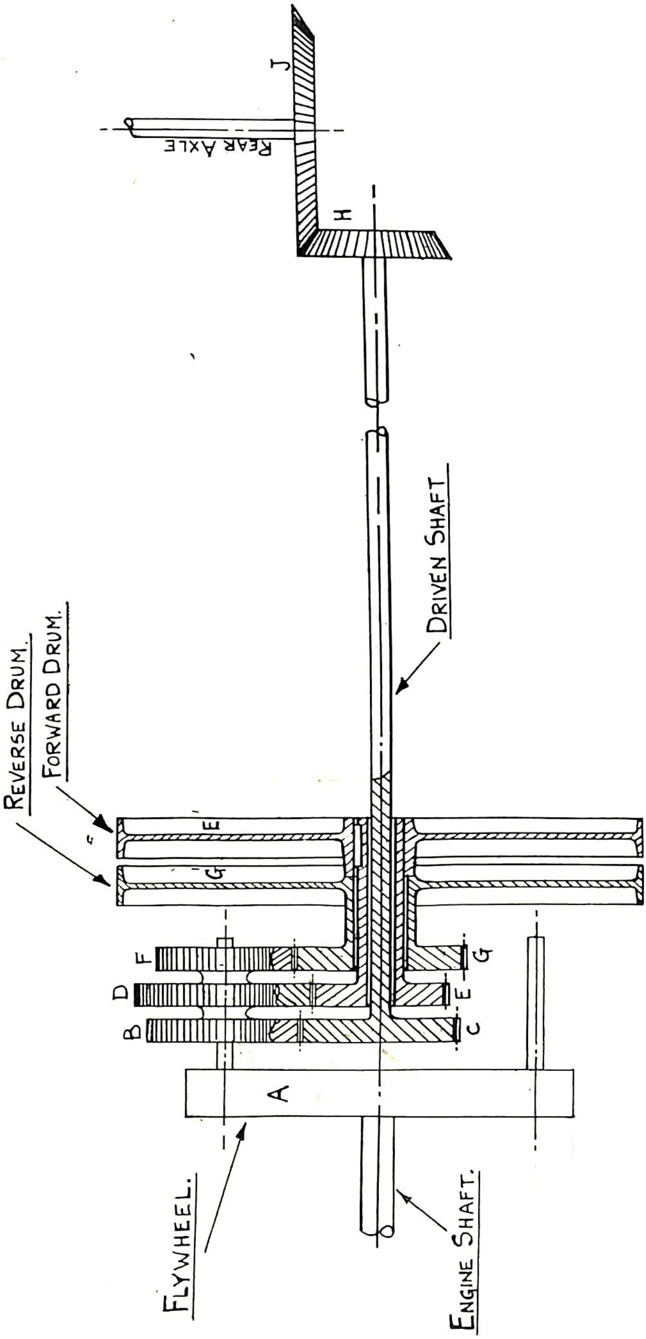

REVERSE DRUM.
FORWARD DRUM.
FLYWHEEL.
ENGINE SHAFT.
DRIVEN SHAFT
REAR AXLE
A
B
C
D
E
F
G
E
G
H
J

Fig. 6.

and the rear axle is not shown as reference will be made to this gear later. D gears with E, which can be fixed by means of a band brake round the forward brake drum E^1. F gears with G, which can be fixed by means of a band brake round the reverse brake drum G^1. The numbers of teeth in the various wheels are :— B—27 teeth ; D—33 teeth ; F—24 teeth ; C—27 teeth ; E—21 teeth ; G—30 teeth ; H—22 teeth ; J—80 teeth. The gear really consists of three distinct trains which are obtained as follows :—

(a) **Top Gear.**

In this gear the wheel B gears with C, which is connected through a clutch with the driven shaft, the forward and reverse brake drums are both free and the gear becomes an ordinary train ABCHJ ; the planet pinion BDF does not revolve on the pin, and the whole gear runs "solid", the flywheel A driving the wheel C at engine speed. So that the velocity ratio of the epicyclic portion of this train is 1/1. Including the bevel train HJ, the overall velocity ratio between the engine and rear axle will be

$$\frac{1}{1} \times \frac{22}{80} = \frac{1}{3{\cdot}63}$$

(b) **Second Gear.**

In this case the forward brake drum E^1 is fixed and the gear becomes an epicyclic train ABCDE, with, of course, the ordinary bevel train HJ at the rear axle.

FIXED	REVS. OF			
↓	A	B, D	C	E
A	0	$+1$	-1	$-\,^{33}/_{21}$
E	$^{33}/_{21}$	$1 + {}^{33}/_{21}$	$^{33}/_{21} - 1$	0
	Driver		Follower	Fixed Wheel

Drawing up the table in the usual way, the first line is obtained by fixing the flywheel or arm A and giving the planet pinion BD one positive revolution. The second line is obtained by subtracting the revolutions of the fixed wheel E, *i.e.* $-\,^{33}/_{21}$ from each item in the first line.

Then velocity ratio of the epicyclic portion of the train

$$= \frac{\text{Revs. of follower C}}{\text{Revs. of driver A}}$$

$$= \frac{^{33}/_{21} - 1}{^{33}/_{21}}$$

$$= \frac{33-21}{33}$$

$$= \frac{1}{2{\cdot}75}$$

Including the bevel train HJ, the overall velocity ratio between the engine and the rear axle will be

$$\frac{1}{2{\cdot}75} \times \frac{22}{80} = \frac{1}{10}$$

(c) **Reverse Gear.**

In this case the forward brake drum E^1 is free and the reverse brake drum G^1 is fixed. The gear then becomes an epicyclic train ABCFG, with the ordinary bevel train HJ at the rear axle. Drawing up the table as before we have :—

FIXED ↓	REVS. OF			
	A	B, F	C	G
A	0	$+1$	-1	$-{}^{24}/_{30}$
G	${}^{24}/_{30}$	$1 + {}^{24}/_{30}$	$-1 + {}^{24}/_{30}$	0
	Driver		Follower	Fixed Wheel

Velocity ratio of the epicyclic portion of the train

$$= \frac{\text{Revs. of follower C}}{\text{Revs. of driver A}}$$

$$= \frac{-1 + {}^{24}/_{30}}{{}^{24}/_{30}}$$

$$= \frac{-30 + 24}{24}$$

$$= -\frac{1}{4}$$

Including the bevel train HJ, the overall velocity ratio between the engine and the rear axle will be

$$= -\frac{1}{4} \times \frac{22}{80}$$

$$= -\frac{1}{14{\cdot}55}$$

Bevel Epicyclic Gears.

In dealing with bevel epicyclic gears, special care must be taken with regard to the direction of rotation and the gear should be viewed from some angle in which none of the wheels is seen "edge on."

Example 4.—Humpage's Gear. Fig. 7.

This is a common type of bevel epicyclic gear frequently used in lathe headstocks.

In the gear shown in Fig. 7,

A is the fixed wheel with 80 teeth.
B with 64 teeth }
C with 30 teeth } is a compound planet pinion.
D is the driven wheel with 50 teeth.
E is the driving wheel with 20 teeth.
F is an arm free to rotate on the axis of the driving shaft.

FIXED ↓	REVS. OF				
	A	B, C	D	E	F
F	$-{}^{64}/_{80}$	$+1$	$-{}^{30}/_{50}$	$+{}^{64}/_{20}$	0
A	0	$1+{}^{64}/_{80}$	$-{}^{30}/_{50}+{}^{64}/_{80}$	${}^{64}/_{20}+{}^{64}/_{80}$	${}^{64}/_{80}$
	Fixed		Follower	Driver	

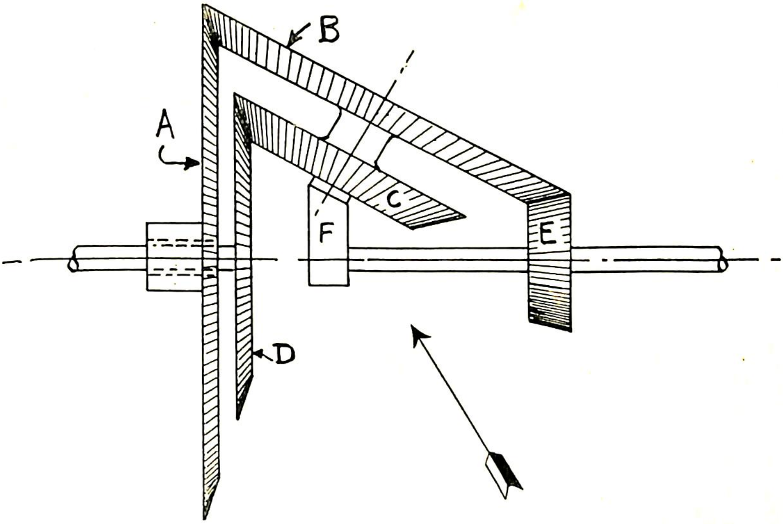


Fig. 7.

To obtain the first line of the table, view the gear *in the direction of the arrow,* fix the arm F and give the planet pinion BC one positive revolution. To obtain the second line of the table, subtract the revolutions of the fixed wheel A, *i.e.* $-{}^{64}/_{80}$ from each item in the first line.

$$\text{Then velocity ratio of gear} = \frac{\text{Revs. of follower D}}{\text{Revs. of driver E}}$$

$$= \frac{-\,{}^{30}/_{50} + {}^{64}/_{80}}{{}^{64}/_{20} + {}^{64}/_{80}}$$

$$= \frac{-\,({}^{30}/_{50} \times {}^{80}/_{64}) + 1}{4 + 1}$$

$$= \frac{{}^{1}/_{4}}{5}$$

$$= \frac{1}{20}$$

The foregoing examples deal only with the particular case of an epicyclic gear having one driver and one fixed wheel, but the method is equally applicable to the *general* case, *i.e.*, the epicyclic gear with *two* drivers and *all wheels free.* The following example will illustrate the procedure to be adopted in dealing with the general case.

Example 5.—Humpage's Gear. Fig. 7.

Consider the bevel gear used in example 4, but let the wheel E run at 400 R.P.M. and the wheel A, instead of being fixed, run at —40 R.P.M., the minus sign indicating that the direction of rotation of A is opposite to that of the driving wheel E. Let it be required to find the speed of the driven wheel D under these conditions.

FIXED ↓	REVS. OF				
	A	B, C	D	E	F
F	$-\,{}^{64}/_{80}$	$+\,1$	$-\,{}^{30}/_{50}$	$+\,{}^{64}/_{20}$	0
All Free	$x - {}^{64}/_{80}$	$x + 1$	$x - {}^{30}/_{50}$	$x + {}^{64}/_{20}$	x

The letters in the table refer to Fig. 7, and the first line of the table is obtained as before by fixing the arm F and giving the planet pinion BC one positive revolution. The second line is obtained by releasing the arm F, giving it x revolutions and adding this quantity x to each item in the first line. To determine the value of the quantity x we are given that the speed of A is —40 R.P.M. and the speed of E is 400 R.P.M.

$$\therefore \quad \frac{\text{Speed of A}}{\text{Speed of E}} = \frac{-40}{400} = -\frac{1}{10} \qquad (1)$$

But from the second line of the table we have

$$\frac{\text{Speed of A}}{\text{Speed of E}} = \frac{x - {}^{64}/_{80}}{x + {}^{64}/_{20}} = -\frac{1}{10} \qquad \text{from (1)}$$

$$\therefore \quad \frac{20\,x - 16}{20\,x + 64} = -\frac{1}{10}$$

$$\therefore \quad 200\,x - 160 = -20\,x - 64$$

$$\therefore \quad 220\,x = 96$$

$$\therefore \quad x = \frac{96}{220}$$

$$= \frac{24}{55}$$

Again from the second line of the table we have

$$\frac{\text{Speed of D}}{\text{Speed of E}} = \frac{x - {}^{30}/_{50}}{x + {}^{64}/_{20}}$$

$$= \frac{{}^{24}/_{55} - {}^{30}/_{50}}{{}^{24}/_{35} + {}^{64}/_{20}}$$

$$= \frac{1 - ({}^{30}/_{50} \times {}^{55}/_{24})}{1 + ({}^{64}/_{20} \times {}^{55}/_{24})}$$

$$= \frac{1 - {}^{33}/_{24}}{1 + {}^{176}/_{24}}$$

$$= -\frac{9}{200}$$

but the speed of E $= 400$ R.P.M.

$$\therefore \quad \text{Speed of D} = -\frac{9}{200} \times 400 \text{ R.P.M.}$$

$$= -\mathbf{18} \text{ R.P.M.}$$

i.e., D runs at 18 R.P.M. in a direction *opposite to that of E*.

As a check on this result, let us compare the speeds of D and A. From the second line of the table we have

$$\frac{\text{Speed of D}}{\text{Speed of A}} = \frac{x - {}^{30}/_{50}}{x - {}^{64}/_{80}}$$

$$= \frac{{}^{24}/_{55} - {}^{30}/_{50}}{{}^{24}/_{55} - {}^{64}/_{80}}$$

$$= \frac{1 - \left({}^{30}/_{50} \times {}^{55}/_{24}\right)}{1 - \left({}^{64}/_{80} \times {}^{55}/_{24}\right)}$$

$$= \frac{1 - {}^{11}/_{8}}{1 - {}^{11}/_{6}}$$

$$= \frac{24-33}{24-44}$$

$$= \frac{9}{20}$$

but the speed of A $= - 40$ R.P.M.

$\therefore$ speed of D $= \frac{9}{20} \times - 40$ R.P.M.

$= -$ **18** R.P.M.

This is the same result as was obtained by comparing the speed of D with the speed of E.

As a last example, let us take a case where the required velocity ratio is known and the numbers of teeth of some of the wheels are unknown.

Example 6.—In the gear shown in Fig. 8, A is a wheel with 60 teeth. The arm D which connects the centres of A and B is required to revolve at 100 R.P.M. in the same direction as wheel C, which runs at 130 R.P.M. when A is fixed. C is concentric with A, and each wheel, except A, is free to revolve on its own axis. The numbers of teeth in the wheels B and C are required for these conditions.

Let the number of teeth in B be N_1 and number of teeth in C be N_2.

Fixed ↓	Revs. of			
	A	B	C	D
D	$-\frac{N_1}{60}$	$+1$	$+\frac{N_1}{N_2}$	0
A	0	$1+\frac{N_1}{60}$	$\frac{N_1}{N_2}+\frac{N_1}{60}$	$\frac{N_1}{60}$

The first line of the table is obtained by fixing the arm D and giving B one positive revolution. The second line is obtained by subtracting the revolutions of the fixed wheel A, *i.e.*, $-\frac{N_1}{60}$ from each item in the first line.

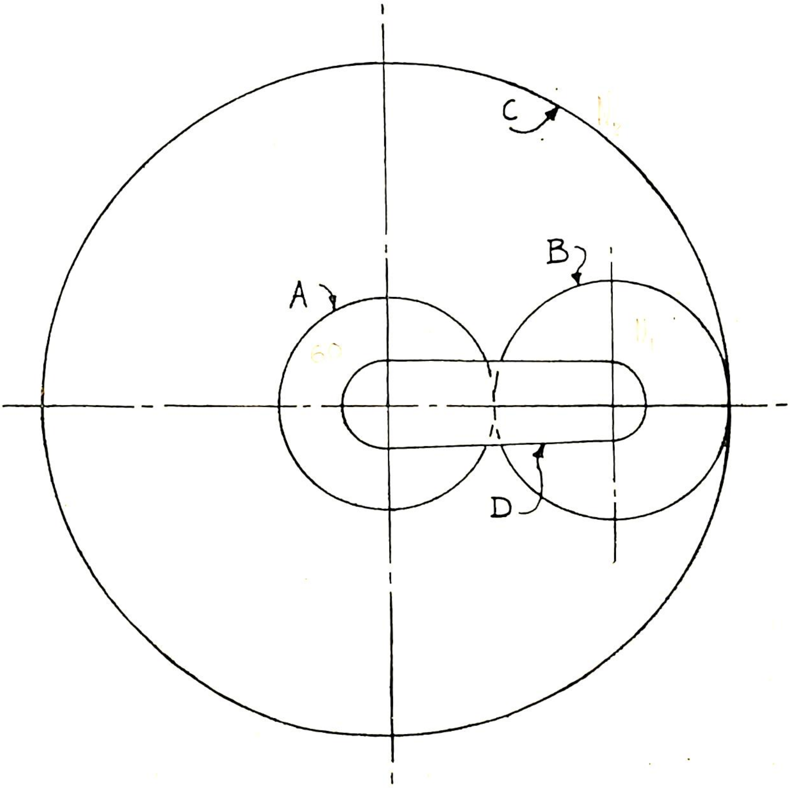


Fig. 8.

Then from the second line of the table we have

$$\frac{\text{Speed of C}}{\text{Speed of D}} = \frac{\dfrac{N_1}{N_2} + \dfrac{N_1}{60}}{\dfrac{N_1}{60}}$$

But $$\frac{\text{Speed of C}}{\text{Speed of D}} = \frac{130}{100} \quad \text{(given)}$$

$$\therefore \quad \frac{\dfrac{N_1}{N_2} + \dfrac{N_1}{60}}{\dfrac{N_1}{60}} = \frac{130}{100}$$

$$\therefore \quad \frac{\dfrac{60}{N_2} + 1}{1} = \frac{130}{100}$$

$$\therefore \quad \frac{60 + N_2}{N_2} = \frac{130}{100}$$

$$\therefore \quad 130\,N_2 = 6000 + 100\,N_2$$

$$\therefore \quad 30\,N_2 = 6000$$

$$\therefore \quad N_2 = \mathbf{200}$$

Again, since the wheels A, B and C are all in gear with each other, they must necessarily have the same pitch of teeth, and

$$\frac{\text{Number of teeth in A}}{2} + \text{number of teeth in B must equal } \frac{\text{Number of teeth in C}}{2}$$

$$\therefore \quad 30 + N_1 = \frac{200}{2}$$

$$\therefore \quad N_1 = \mathbf{70}$$

that is, for the required conditions, B must have 70 teeth and C must have 200 teeth.

Tooth Loads and Friction Losses.

With accurately machined spur gears the friction loss at each pair of wheels may be taken as one per cent. of the power transmitted between the wheels. In practice this loss in ordinary fixed centre trains is so small that it may be ignored, but with epicyclic gears very high tooth loads are generated at high pitch line velocities and the loss due to friction becomes appreciable and must be taken into account when considering the efficiency of the gear as a power transmitting medium. The loss due to friction at each pair of wheels is usually taken as one per cent. of the *virtual* work transmitted between them. The method adopted for calculating tooth loads and friction losses will be illustrated in the following examples. In choosing the examples, two main types of epicyclic gearing have been chosen, the first in which the efficiency compares very favourably with that of an ordinary fixed centre train and the second in which the efficiency, especially for high velocity ratios, is low compared with that of an ordinary fixed centre train.

Example 1.—Fig. 9 (a) shows a simple type of epicyclic gear in which the wheel A is the driver keyed to the driving shaft. An arm B keyed to the driven shaft carries a planet pinion C which gears with A and also with the fixed wheel D. Let the wheel A run at 1000 R.P.M. and transmit 3 H.P. to *driven* shaft and let it be

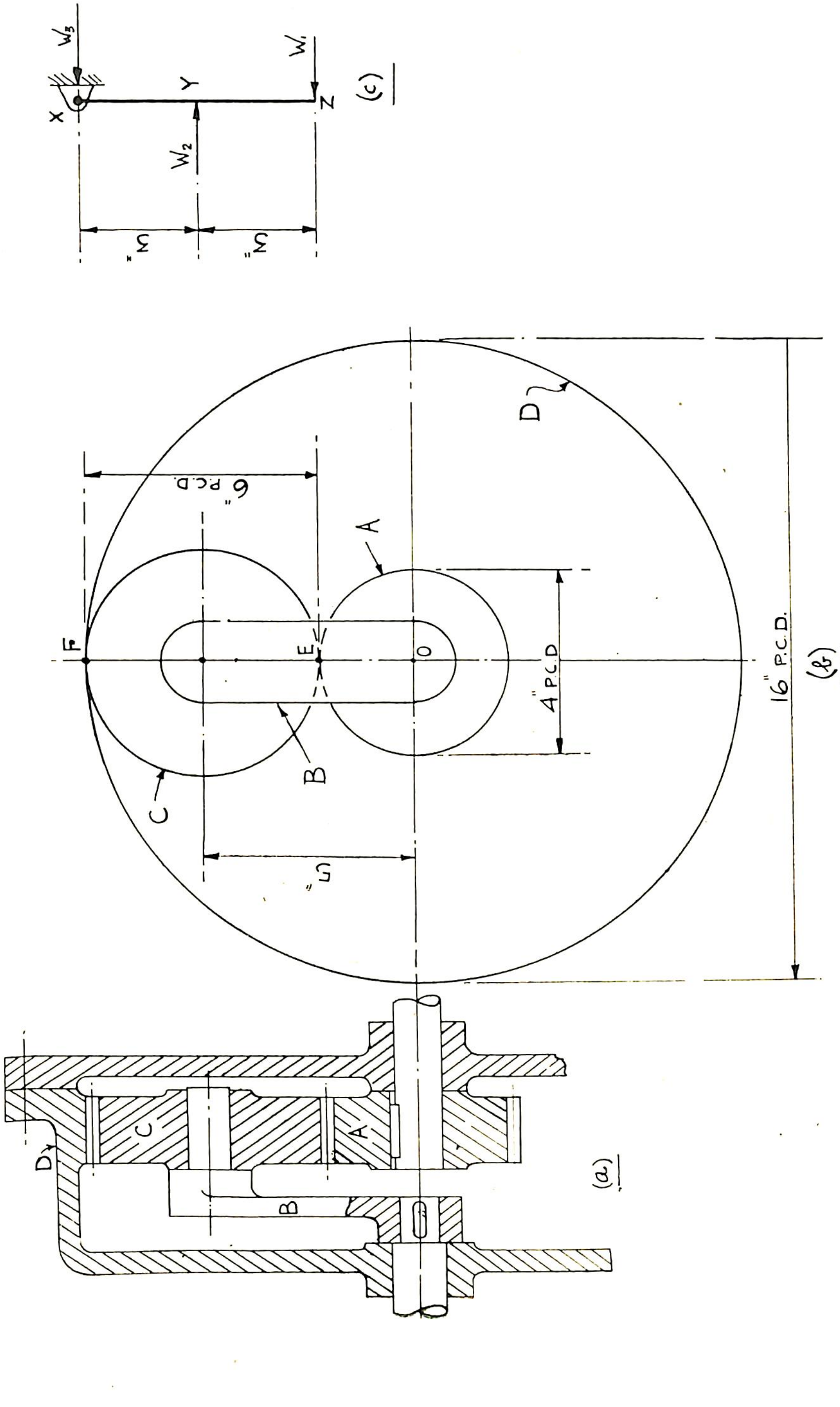


Fig. 9.

required to find the velocity ratio, tooth loads, friction losses and efficiency of the gear. Fig. 9 (a) shows the arrangement of the gear. Fig. 9 (b) shows diagrammatically the pitch circles of the wheels. To make the example more definite, let us assume that the pitch circle diameters of the wheels are A—4″, C—6″ and D—16″. The radius of the arm will then be 5″.

Velocity Ratio.

Drawing up the table in the usual fashion we have

FIXED	REVS. OF			
↓	A	B	C	D
B	$-{}^{6}/_{4}$	0	$+1$	$+{}^{6}/_{16}$
D	$-{}^{6}/_{4}-{}^{6}/_{16}$	$-{}^{6}/_{16}$	$1-{}^{6}/_{16}$	0

$$\text{Velocity ratio} = \frac{\text{Revs. of B}}{\text{Revs. of A}} = \frac{-{}^{6}/_{16}}{-{}^{6}/_{4}-{}^{6}/_{16}}$$

$$= \frac{\frac{3}{8}}{1\frac{7}{8}}$$

$$= \frac{1}{5}$$

$$\therefore \quad \text{Speed of B} = \tfrac{1}{5} \times 1000 \text{ R.P.M.}$$
$$= 200 \text{ R.P.M.}$$

Tooth Loads.

Let W_1 be the tooth load between the wheels A and C.

W_2 be the force applied to the driven shaft by the arm B, the point of application being the centre of the planet pinion C.

W_3 be the tooth load between the wheels C and D.

Then the system so far as loads are concerned may be compared to a simple lever XYZ, as shown in Fig. 9 (c), the pitch point of the fixed wheel D being the fulcrum.

H.P. transmitted by wheel A = 3.

Pitch line velocity of wheel A = pitch circumference × speed in R.P.M.

$$= \frac{\pi \times 4}{12} \times 1000$$

$$= \frac{1000\,\pi}{3} \text{ ft. per min.}$$

And from the relation, $$\text{H.P.} = \frac{\text{Tooth load} \times \text{Pitch line velocity}}{33{,}000}$$

$$\text{we have } 3 = \frac{W_1 \times 1000\pi}{33{,}000 \times 3}$$

$$\therefore \quad W_1 = \mathbf{94{\cdot}6\ lb.}$$

Taking moments about the fulcrum X we have

$$W_2 \times 3 = W_1 \times 6$$
$$\therefore \quad W_2 = 94{\cdot}6 \times 2$$
$$= \mathbf{189{\cdot}2\ lb.}$$

Again, for equilibrium, $W_1 + W_3 = W_2$

$$\therefore \quad W_3 = W_2 - W_1$$
$$= 189{\cdot}2 - 94{\cdot}6$$
$$= \mathbf{94{\cdot}6\ lb.}$$

As a check on these tooth loads, the arm B rotates at 200 R.P.M. and has to transmit 3 H.P. with the load W_2 acting at a radius of 5″ from the centre of the shaft. This will give W_2 a velocity of

$$\frac{2\pi \times 5}{12} \times 200 \text{ ft. per min.}$$

$$\textit{i.e.,} \quad \frac{1000\pi}{6} \text{ ft. per min.}$$

And from the relation, $$\text{H.P.} = \frac{\text{Load} \times \text{velocity}}{33{,}000}$$

$$\text{we have } 3 = \frac{W_2 \times 1000\pi}{6 \times 33{,}000}$$

$$\therefore \quad W_2 = \frac{18 \times 33}{\pi}$$
$$= \mathbf{189{\cdot}2\ lb.}$$

which is the same value as that obtained from the lever analogy.

Friction Losses and Efficiency of Gear.

As the friction losses at each pair of gears in an epicyclic train depend on the virtual work done between them, it is a necessary preliminary to the calculation of friction losses to calculate the virtual work done at each pair of wheels. Consider first the wheels A and C. Then, since the pitch line velocity of wheel A is $\frac{1000\pi}{3}$ ft. per min., a point E on the pitch surface of wheel A will have a velocity of $\frac{1000\,\pi}{3}$ ft. per min. = 1047 ft. per min., clockwise relative to the axes of the shafts. This would be the velocity of transmission if the train were an ordinary fixed centre one. But in virtue of the fact that the planet pinion C is attached to the arm B, a point E on the pitch surface of C, at a radius of 2″ from O, will, since B runs at 200 R.P.M., have a velocity of $\frac{2\,\pi \times 2}{12} \times 200$ ft. per min. = 209 ft. per min. relative to the axis of the shafts. That is, the point of contact E on the wheel A has a velocity of 1047 ft. per min. relative to the axis of the shafts in virtue of the motion of wheel A and a point of contact E on the wheel C has a velocity of 209 ft. per min. relative to the axis of the shafts in virtue of the motion of the arm B. Since these two velocities are in the same direction, the relative velocity of point of contact E on the wheel A to point of contact E on the wheel C will be given by the difference of these two velocities, *i.e.*, by 1047 − 209 = 838 ft. per min. The load between the gears is 94·6 lb. and the virtual work done will be given by 94·6 × 838 = 79,200 ft. lbs. per min., and, taking the friction loss as 1% of the virtual work, the friction loss in the train A-C will be 792 ft. lb. per min.

Consider now the wheels C and D. Since the wheel D is fixed, a point F on its surface will have no velocity relative to the axis of the shafts. But, in virtue of the fact that the planet pinion C is attached to the arm B, a point F on the pitch surface of the wheel C, at a radius of 8″ from O, will, since B runs at 200 R.P.M. have a velocity of $\frac{2\,\pi \times 8}{12} \times 200$ ft. per min. = 838 ft. per min. clockwise relative to the axis of the shafts. Hence the relative velocity of point of contact F on pinion C to point of contact F on pinion D = 838 − 0 = 838 ft. per min. The load between the gears is 94·6 lb. and the virtual work done will be given by 94·6 × 838 = 79,200 ft. lb. per min., and, taking the friction loss at 1% of the virtual work, the friction loss in the train C-D will be 792 ft. lb. per min.

Total friction loss = (792 + 792) ft. lb. per min.
= 1584 ft. lb. per min.
= 0·048 H.P.

That is, to transmit 3 H.P. to B, 3·048 H.P. must be supplied *at A* giving an efficiency of $\frac{3}{3{\cdot}048}$ = 0·984 or 98·4%, which would compare very favourably with the efficiency of an ordinary train of gears giving the same velocity ratio.

Example 2.—Fig. 10 (a) shows a type of epicyclic gear with a compound planet pinion. The arm A is the driver keyed to the driving shaft B, C is a compound planet pinion revolving freely on a pin carried by the arm A, C gearing with the fixed wheel D, and B gearing with the wheel E, which is keyed to the driven shaft. Let the arm A run at 1000 R.P.M. and transmit 3 H.P. to the *driven* shaft and let it be required to find the velocity ratio, tooth loads, friction losses and efficiency of the gear. Fig. 10 (b) shows diagrammatically the pitch circles of the wheels. To make the example more definite, let us assume that the pitch circle diameters of the wheels are B—3″, C—4″, D—9″ and E—10″. The radius of the arm A will then be $6\frac{1}{2}$″.

Velocity Ratio.

Drawing up the table in the usual fashion we have

FIXED	REVS. OF			
↓	A	B, C	D	E
A	0	$+\ 1$	$-\ {}^{4}/_{9}$	$-\ {}^{3}/_{10}$
D	${}^{4}/_{9}$	$1 + {}^{4}/_{9}$	0	$-{}^{3}/_{10} + {}^{4}/_{9}$

$$\text{Velocity ratio} = \frac{\text{Revs. of E}}{\text{Revs. of A}} = \frac{-\ {}^{3}/_{10} + {}^{4}/_{9}}{{}^{4}/_{9}}$$

$$= \frac{-\ ({}^{3}/_{10} \times {}^{9}/_{4}) + 1}{1}$$

$$= \frac{1 - {}^{27}/_{40}}{1}$$

$$= \frac{13}{40}$$

$$\therefore \quad \text{Speed of E} = {}^{13}/_{40} \times 1000 \text{ R.P.M.}$$

$$= 325 \text{ R.P.M.}$$

Tooth Loads.

Let W_1 be the force applied to the axis of the planet pinions B, C by the arm A keyed to the driving shaft.
W_2 be the tooth load between the wheels B and E.
W_3 be the tooth load between the wheels C and D.

Then the system as far as loads are concerned may be compared to a simple lever XYZ, as shown in Fig. 10 (c), the pitch point of the fixed wheel D being the fulcrum.

H.P. transmitted by the arm A = 3.

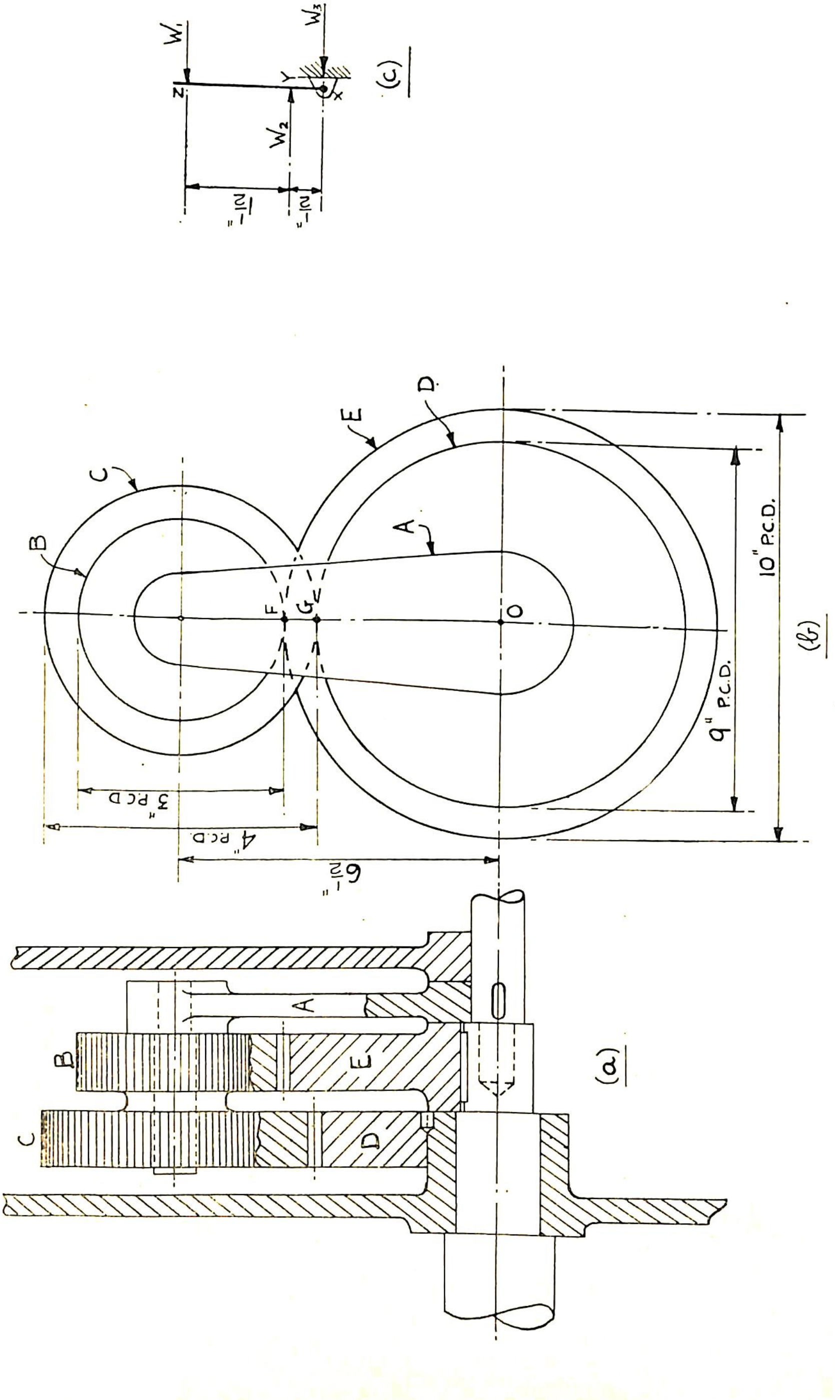


Fig. 10.

Velocity of point of application of load W_1 = circumference of circle of $6\frac{1}{2}''$ rad. × speed of arm in R.P.M.

$$= \frac{\pi \times 13}{12} \times 1000 \text{ ft. per min.}$$

And from the relation,

$$\text{H.P. transmitted} = \frac{\text{load} \times \text{velocity}}{33{,}000}$$

$$\text{we have} \quad 3 = \frac{W_1 \times \pi \times 13 \times 1000}{12 \times 33{,}000}$$

$$\therefore \quad W_1 = \frac{3 \times 12 \times 33{,}000}{\pi \times 13 \times 1000}$$

$$= \mathbf{29{\cdot}1 \ lb.}$$

Taking moments about the fulcrum X we have

$$W_2 \times \tfrac{1}{2} = W_1 \times 2$$

$$\therefore \quad W_2 = W_1 \times 4$$

$$= 29{\cdot}1 \times 4$$

$$= \mathbf{116{\cdot}4 \ lb.}$$

$$\text{Again, for equilibrium,} \quad W_1 + W_3 = W_2$$

$$\therefore \quad W_3 = W_2 - W_1$$

$$= 116{\cdot}4 - 29{\cdot}1$$

$$= \mathbf{87{\cdot}3 \ lb.}$$

As a check on these tooth loads, the wheel E rotates at 325 R.P.M. and has to transmit 3 H.P. with the load W_2 acting at a pitch circle of radius 5″. This will give W_2 a velocity of $\frac{2\pi 5}{12} \times 325$ ft. per min. and from the relation $\text{H.P.} = \frac{\text{load} \times \text{velocity}}{33{,}000}$

$$\text{we have} \quad 3 = \frac{W_2 \times 2\pi \times 5 \times 325}{12 \times 33{,}000}$$

$$\therefore \quad W_2 = \frac{3 \times 12 \times 33{,}000}{2 \times \pi \times 5 \times 325}$$

$$= \mathbf{116{\cdot}4 \ lb.}$$

which is the same value as that obtained by the lever analogy.

Friction Losses and Efficiency of Gear.

Consider first the wheels B and E. The pitch circumference of the wheel E $= \frac{2\pi \times 5}{12}$ ft. and the speed of E is 325 R.P.M. Then the pitch line velocity of the wheel E $= \frac{2\pi \times 5}{12} \times 325$ ft. per min. $= \frac{3250\pi}{12}$ ft. per min. Therefore a point F on the pitch surface of the wheel E will have a velocity of $\frac{3250\pi}{12} = 852$ ft. per min. clockwise relative to the axis of the shafts. This would be the velocity of transmission if the gear were an ordinary fixed centre train. But, in virtue of the fact that the planet pinion B is attached to the arm A, a point F on the pitch surface of B at a radius of 5″ from O will, since A runs at 1000 R.P.M., have a velocity of $\frac{2\pi 5}{12} \times 1000$ ft. per min. $= 2618$ ft. per min. clockwise relative to the axis of the shafts. That is, a point of contact F on wheel B has a velocity of 2618 ft. per min. relative to the axis of the shafts in virtue of the motion of the arm A and a point of contact F on wheel E has a velocity of 852 ft. per min. relative to the axis of the shafts in virtue of the motion of E. Since these two velocities are in the same direction the relative velocity of point of contact F on wheel B to point of contact F on wheel E will be given by the difference of the two velocities, *i.e.*, by $2618 - 852 = 1766$ ft. per min. The load between the gears is 116·4 lb. and the virtual work done will be given by $116{\cdot}4 \times 1766 = 205{,}600$ ft. lb. per min. and, taking the friction loss as 1% of the virtual work, the friction loss in the train B-E will be 2056 ft. lb. per min. Consider now the wheels C and D. Since the wheel D is fixed, a point G on its pitch surface will have no velocity relative to the axis of the shafts. But, in virtue of the fact that the planet pinion C is attached to the arm A, a point G in the pitch surface of C, at a radius of $4\frac{1}{2}$″ from O, will, since A runs at 1000 R.P.M., have a velocity of $\frac{2\pi \times 4{\cdot}5}{12} \times 1000 =$ 2357 ft. per min. clockwise relative to the axis of the shaft. Hence the relative velocity of the point of contact G on pinion C to the point of contact G on the wheel D $= 2357 - 0 = 2357$ ft. per min. The load between the gears is 87·3 lb. and the virtual work done will be given by $87{\cdot}3 \times 2357 = 205{,}600$ ft. lb. per min. and, taking the friction loss as 1% of the virtual work done, the friction loss in the train C-D will be 2056 ft. lb. per min.

Total friction loss = (2056 + 2056) ft. lb. per min.
= 4112 ft. lb. per min.
= 0·125 H.P.

That is, to transmit 3 H.P. *to E*, 3·125 H.P. must be supplied *at*

A, giving an efficiency of $\frac{3}{3{\cdot}125} = 0{\cdot}96$ or 96%. This efficiency, while not quite so high as that obtained with the gear in example 1, would still compare very favourably with an ordinary train of gears to give the same velocity ratio. Unfortunately the efficiency of an epicyclic train of this type falls off rather badly when the reduction ratio is increased. Example 3 will illustrate this point.

Example 3.—With the same type of gear as in example 2, let the pitch circle diameters of the wheels be B—3·9″, C—4″, D—10″ and E—10·1″. The radius of the arm A will then be 7″. As before, let the driving arm A run at 1000 R.P.M. and transmit 3 H.P. to the *driven* wheel E and let it be required to find the velocity ratio, tooth loads, friction losses and efficiency of the gear. Fig. 11 (b) shows diagrammatically the pitch circles of the wheels.

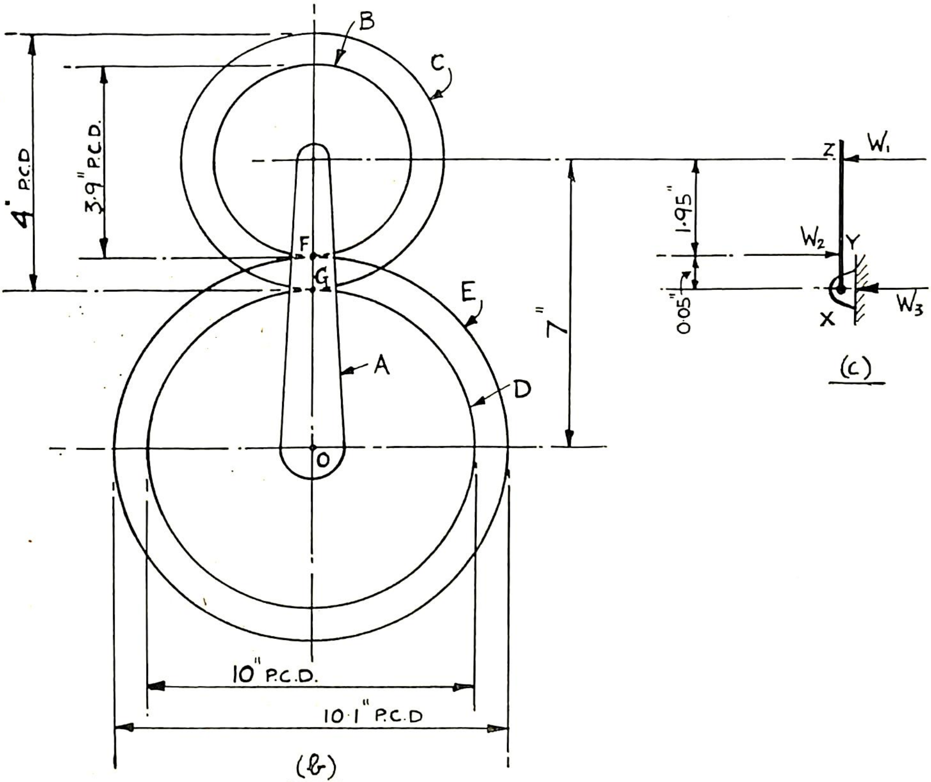


Fig. 11.

the pitch line velocity of the wheel E $= \frac{2\pi \times 5}{12} \times 325$ ft. per min. $= \frac{3250\pi}{12}$ ft. per min. Therefore a point F on the pitch surface of the wheel E will have a velocity of $\frac{3250\pi}{12} = 852$ ft. per min. clockwise relative to the axis of the shafts. This would be the velocity of transmission if the gear were an ordinary fixed centre train. But, in virtue of the fact that the planet pinion B is attached to the arm A, a point F on the pitch surface of B at a radius of 5″ from O will, since A runs at 1000 R.P.M., have a velocity of $\frac{2\pi 5}{12} \times 1000$ ft. per min. $= 2618$ ft. per min. clockwise relative to the axis of the shafts. That is, a point of contact F on wheel B has a velocity of 2618 ft. per min. relative to the axis of the shafts in virtue of the motion of the arm A and a point of contact F on wheel E has a velocity of 852 ft. per min. relative to the axis of the shafts in virtue of the motion of E. Since these two velocities are in the same direction the relative velocity of point of contact F on wheel B to point of contact F on wheel E will be given by the difference of the two velocities, *i.e.*, by $2618 - 852 = 1766$ ft. per min. The load between the gears is 116·4 lb. and the virtual work done will be given by $116{\cdot}4 \times 1766 = 205{,}600$ ft. lb. per min. and, taking the friction loss as 1% of the virtual work, the friction loss in the train B-E will be 2056 ft. lb. per min. Consider now the wheels C and D. Since the wheel D is fixed, a point G on its pitch surface will have no velocity relative to the axis of the shafts. But, in virtue of the fact that the planet pinion C is attached to the arm A, a point G in the pitch surface of C, at a radius of $4\frac{1}{2}$″ from O, will, since A runs at 1000 R.P.M., have a velocity of $\frac{2\pi \times 4{\cdot}5}{12} \times 1000 =$ 2357 ft. per min. clockwise relative to the axis of the shaft. Hence the relative velocity of the point of contact G on pinion C to the point of contact G on the wheel D $= 2357 - 0 = 2357$ ft. per min. The load between the gears is 87·3 lb. and the virtual work done will be given by $87{\cdot}3 \times 2357 = 205{,}600$ ft. lb. per min. and, taking the friction loss as 1% of the virtual work done, the friction loss in the train C-D will be 2056 ft. lb. per min.

Total friction loss = (2056 + 2056) ft. lb. per min.
= 4112 ft. lb. per min.
= 0·125 H.P.

That is, to transmit 3 H.P. *to E*, 3·125 H.P. must be supplied *at*

A, giving an efficiency of $\dfrac{3}{3 \cdot 125} = 0 \cdot 96$ or 96%. This efficiency, while not quite so high as that obtained with the gear in example 1, would still compare very favourably with an ordinary train of gears to give the same velocity ratio. Unfortunately the efficiency of an epicyclic train of this type falls off rather badly when the reduction ratio is increased. Example 3 will illustrate this point.

Example 3.—With the same type of gear as in example 2, let the pitch circle diameters of the wheels be B—3·9″, C—4″, D—10″ and E—10·1″. The radius of the arm A will then be 7″. As before, let the driving arm A run at 1000 R.P.M. and transmit 3 H.P. to the *driven* wheel E and let it be required to find the velocity ratio, tooth loads, friction losses and efficiency of the gear. Fig. 11 (b) shows diagrammatically the pitch circles of the wheels.

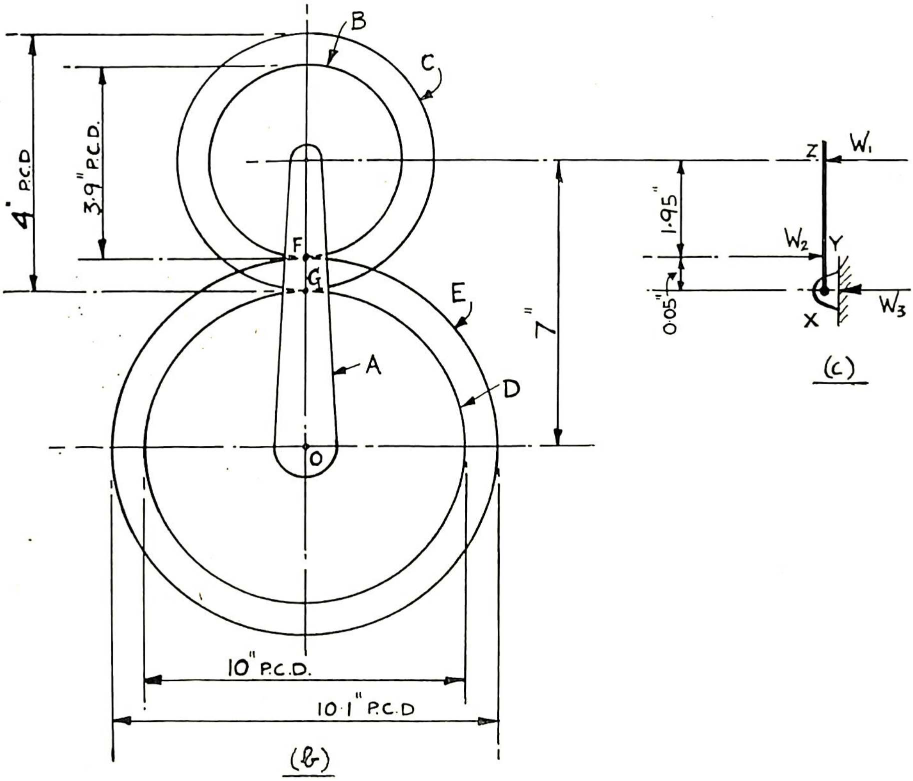


Fig. 11.

Velocity Ratio.

Drawing up the table in the usual fashion, we have

FIXED ↓	REVS. OF			
	A	B, C	D	E
A	0	$+1$	$-\frac{4}{10}$	$-\frac{3{\cdot}9}{10{\cdot}1}$
D	$\frac{4}{10}$	$1+\frac{4}{10}$	0	$\frac{4}{10}-\frac{3{\cdot}9}{10{\cdot}1}$

$$\text{Velocity ratio} = \frac{\text{Revs. of E}}{\text{Revs. of A}} = \frac{\frac{4}{10}-\frac{3{\cdot}9}{10{\cdot}1}}{\frac{4}{10}}$$

$$= \frac{1-\left(\frac{3{\cdot}9}{10{\cdot}1}\times\frac{10}{4}\right)}{1}$$

$$= 1-\frac{39}{40{\cdot}4}$$

$$= \frac{1{\cdot}4}{40{\cdot}4}$$

$$\therefore \quad \text{Speed of E} = \frac{1{\cdot}4}{40{\cdot}4}\times 1000$$

$$= 34{\cdot}65 \text{ R.P.M.}$$

Tooth Loads.

Let W_1 be the force applied to the axis of the planet pinions B, C by the arm A keyed to the driving shaft.

Let W_2 be the tooth load between the wheels B and E.

Let W_3 be the tooth load between the wheels C and D.

Then the system as far as loads are concerned may be compared to a simple lever XYZ, as shown in Fig. 11 (c), the pitch point of the fixed wheel D being the fulcrum.

H.P. transmitted by the arm A = 3.

Velocity of point of application of the load W_1 = circumference of circle of 7″ rad. × speed of arm in R.P.M.

$$= \frac{\pi \times 14}{12} \times 1000 \text{ ft. per min.}$$

And from the relation,

$$\text{H.P. transmitted} = \frac{\text{load} \times \text{velocity}}{33{,}000}$$

$$\text{we have} \quad 3 = \frac{W_1 \times \pi \times 14 \times 1000}{12 \times 33{,}000}$$

$$\therefore \quad W_1 = \frac{3 \times 12 \times 33{,}000}{\pi \times 14 \times 1000}$$

$$= \mathbf{27\ lb.}$$

Taking moments about the fulcrum X, we have

$$W_2 \times 0{\cdot}05 = W_1 \times 2$$

$$\therefore \quad W_2 = W_1 \times 40$$

$$= 27 \times 40$$

$$= \mathbf{1080\ lb.}$$

Again, for equilibrium, $W_1 + W_3 = W_2$

$$\therefore \quad W_3 = W_2 - W_1$$

$$= 1080 - 27$$

$$= \mathbf{1053\ lb.}$$

As a check on these results, the wheel E rotates at 34·65 R.P.M. and has to transmit 3 H.P. with the load W_2 acting at the pitch circle radius of 5·05″. This will give W_2 a velocity of $\frac{2\pi \times 5{\cdot}05}{12} \times 34{\cdot}65$ ft. per min., and from the relation

$$\text{H.P.} = \frac{\text{load} \times \text{velocity}}{33{,}000}$$

$$\text{we have} \quad 3 = \frac{W_2 \times 2\pi \times 5{\cdot}05 \times 34{\cdot}65}{33{,}000 \times 12}$$

$$\therefore \quad W_2 = \frac{3 \times 33{,}000 \times 12}{2\pi \times 5{\cdot}05 \times 34{\cdot}65}$$

$$= \mathbf{1080\ lb.}$$

which is the same value as that obtained by the lever analogy.

Friction Losses and Efficiency of Gear.

Consider first the wheels B and E. The pitch circumference of the wheel E $= \frac{\pi \times 10{\cdot}1}{12}$ ft. and the speed of E is 34·65 R.P.M.

Then the pitch line velocity of the wheel E $= \frac{\pi \times 10{\cdot}1}{12} \times 34{\cdot}65$ ft. per min. = 92 ft. per min. clockwise relative to the axis of the shafts. This would be the velocity of transmission if the gear were an ordinary fixed centre train. But, in virtue of the fact that the planet pinion B is attached to the arm A, a point F on the pitch surface of B at a radius of 5·05″ from O will, since A runs at 1000 R.P.M., have a velocity of $\frac{2\,\pi \times 5{\cdot}05 \times 1000}{12}$ ft. per min. =2641 ft. per min. clockwise relative to the axis of the shafts. That is, a point of contact F on the wheel B has a velocity of 2641 ft. per min. relative to the axis of the shafts in virtue of the motion of the arm A and a point of contact F on the wheel E has a velocity of 92 ft. per min. relative to the axis of the shafts in virtue of the motion of E. Since these two velocities are in the same direction, the relative velocity of the point of contact F on the wheel B to the point of contact F on the wheel E will be given by the difference of the two velocities, *i.e.*, by 2641 − 92 = 2549 ft. per min. The load between the gears is 1080 lb. and the virtual work done will be given by 1080 × 2549 ft. lb. per min. = 2,753,000 ft. lb. per min. and taking the friction loss as 1% of the virtual work, the friction loss in the train B-E will be 27,530 ft. lb. per min.

Consider now the wheels C and D. Since the wheel D is fixed, a point G on its pitch surface will have no velocity relative to the axis of the shafts. But, in virtue of the fact that the planet pinion C is attached to the arm A, a point G on the pitch surface of C at a radius of 5″ from 0 will, since A runs at 1000 R.P.M., have a velocity of $\frac{2\,\pi \times 5 \times 1000}{12} = 2618$ ft. per min. clockwise relative to the axis of the shafts. Then the relative velocity of the point of contact G on the pinion C to the point of contact G on the wheel D = 2618 − 0 = 2618 ft. per min. The load between the gears is 1053 lb. and the virtual work done will be given by 1053 × 2618 = 2,757,000 ft. lb. per min., and, taking the friction losses as 1% of the virtual work done, the friction loss in train C - D will be 27,570 ft. lb. per min.

Total friction loss = (27,530 + 27,570) ft. lb. per min.
= 55,100 ft. lb. per min.
= 1·67 H.P.

That is, to transmit 3 H.P. *to E*, 4·67 H.P. must be supplied *at A*, giving an efficiency of $\frac{3}{4{\cdot}67} = 0{\cdot}64$ or 64%, which is very low for spur gear transmission.

Applications of Epicyclic Gearing.

Having disposed of the theoretical principles involved in the calculations of velocity ratios, tooth loads and efficiencies for epicyclic gearing, let us now consider in detail a few of the applications of this type of gearing to modern engineering practice.

Cyclometer.

This is a practical example of the reverted train and is used for recording the distance travelled by an ordinary pedal bicycle. The whole gear is remarkably compact and is usually enclosed in a cylinder of less than 1″ in diameter. Fig. 12 shows diagrammatically the arrangement of the gear.

C and D are concentric annular wheels with centre O, C being fixed and D free to revolve about O. The arm OP also revolves freely about O and carries the compound planet pinion A, B, which is free to revolve about P, A gearing with the fixed wheel C and B gearing with D. The numbers of teeth in the various wheels are A—21 teeth, B—23 teeth, C—24 teeth and D—26 teeth. The arm OP is the driver and the annular wheel D is the follower.

Drawing up the table in the usual fashion, we have

FIXED ↓	REVS. OF			
	A, B	C	D	OP
O P	$+ 1$	$+ {}^{21}/_{24}$	$+ {}^{23}/_{26}$	0
C	$1 - {}^{21}/_{24}$	0	${}^{23}/_{26} - {}^{21}/_{24}$	$- {}^{21}/_{24}$

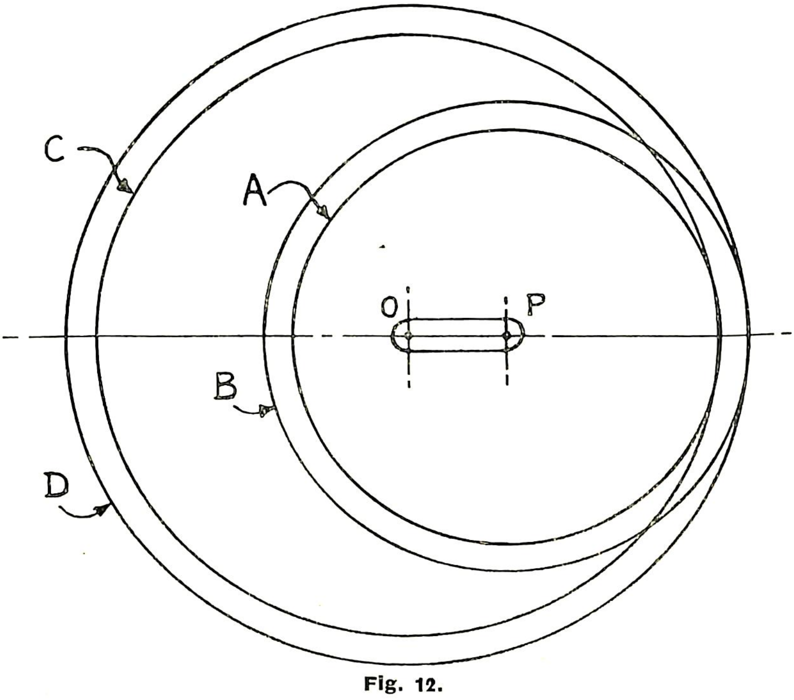


Fig. 12.

$$\text{Velocity ratio} = \frac{\text{Revs. of D}}{\text{Revs. of OP}} = \frac{{}^{23}/_{26} - {}^{21}/_{24}}{-\,{}^{21}/_{24}}$$

$$= \frac{({}^{23}/_{26} \times {}^{24}/_{21}) - 1}{-\,1}$$

$$= \frac{{}^{552}/_{546} - 1}{-\,1}$$

$$= -\,\frac{6}{546}$$

$$= -\,\frac{1}{91}$$

On the bicycle the arm OP is made integral with a star wheel, which has eight teeth. A striker fixed to a spoke of the cycle wheel rotates with the cycle wheel and is arranged in such a manner that it engages with the teeth of the star wheel and causes the star wheel to revolve completely once in eight revolutions of the cycle wheel. This part of the gear is not shown in Fig. 12. With a cycle wheel of 28″ diameter, the circumference of the wheel will be $\frac{\pi \times 28}{12}$ feet and one revolution of OP will correspond to a distance travelled of $\frac{\pi \times 28}{12} \times 8$ feet, since OP makes one revolution for eight revolutions of the wheel. Again, the velocity ratio of D to OP is $\frac{1}{91}$. Therefore one revolution of D will correspond to a distance travelled of $\frac{\pi \times 28 \times 8 \times 91}{12}$ feet.

$$= 5330 \text{ ft.}$$
$$= 1{\cdot}01 \text{ miles.}$$

Since the radius of the tyre at the point of contact with the road will vary with the degree of inflation and with the weight of the rider, it may easily vary from the value of 14″ by 1%. Hence no serious error will be involved if one revolution of D is taken to correspond to a distance travelled of 1 mile instead of 1·01 miles. This is actually assumed and the outside of the wheel D is engraved with the numbers 1, 2, 3 . . . . 9, 0, each number being viewed through a window in the cylinder in which the gear is enclosed, only one figure being visible at a time. Hence a movement of one figure to the next will indicate that one-tenth of a mile has been traversed.

Driving Pulley with Epicyclic Reverse Gear.

There are many applications of epicyclic gearing to machine tools, but one which comes very readily to the mind of the author is that fitted to the driving pulleys of turret lathes. This gear provides three positions—neutral, forward and reverse—all arranged inside one driving pulley. The principle of the gear is outlined diagrammatically in Fig. 13. The driving pulley B is made in halves and runs freely on the driving shaft A. The pulley may be fixed rigidly to A by means of a radial plunger P which operates an expanding brake. Movement of the rod R to the right from the position shown, pushes out the plunger P and brings the expanding brake into contact with the inner surface of the pulley at Z. This causes the pulley and shaft to rotate as one, giving the forward position of the gear. Inside the driving pulley B, a compound planet pinion C, D runs freely on a pin supported by the flange of the pulley and an internal plate G. C gears with the wheel E which is rigidly fixed to the headstock. D gears with a wheel F which runs freely on the driving shaft A, but which may be fixed to A by means of another plunger Q operating an expanding brake which comes into action when the rod R is moved to the left from the position shown. The reverse gear is brought into action by moving the rod R to the left, thereby fixing the wheel F to the driving shaft A, the drive passing from the pulley to the shaft through the gears C, D, E and F. The number of teeth in the various wheels are C—23 teeth, D—32 teeth, E—34 teeth and F—25 teeth. Drawing up the table in the usual fashion we have :—

FIXED ↓	REVS. OF			
	C, D	E	F	B
B	$+1$	$-{}^{23}/_{34}$	$-{}^{32}/_{25}$	0
E	$1+{}^{23}/_{34}$	0	$-{}^{32}/_{25}+{}^{23}/_{34}$	$+{}^{23}/_{34}$

Velocity ratio of gear

$$= \frac{\text{Revs. of follower F}}{\text{Revs. of driver B}}$$

$$= \frac{-{}^{32}/_{25} + {}^{23}/_{34}}{{}^{23}/_{34}}$$

$$= \frac{-({}^{32}/_{25} \times {}^{34}/_{23}) + 1}{1}$$

$$= \frac{-1{\cdot}89 + 1}{1} = \frac{-0{\cdot}89}{1} = \frac{-1}{1{\cdot}125}$$

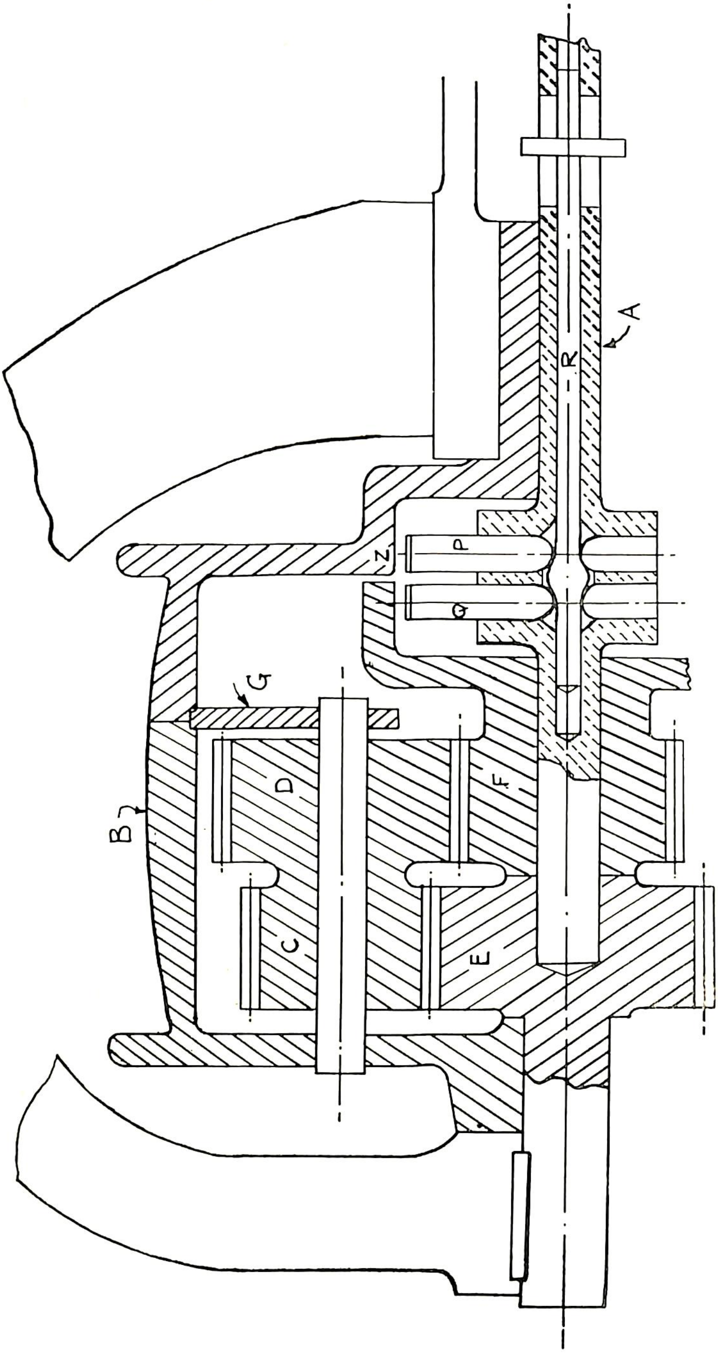


Fig. 13.

So that in reverse gear, driving pulley B makes $1\frac{1}{8}$ revs. in one direction, while the driving shaft A makes 1 revolution in the opposite direction. The neutral position is obtained with the rod R in the position shown in Fig. 13, plungers P and Q being both out of action and the driving pulley running idly on the shaft.

The Wilson Gearbox.

Perhaps no application of epicyclic gearing has aroused so much interest in recent years as this remarkable gearbox arranged to give a neutral position, four forward gears and a reverse gear. The inventor, Major W. G. Wilson, C.M.G., read a comprehensive paper on the action of the gearbox before the I.A.E. (Vol. XXVI., p. 216) and readers of this pamphlet are referred to this paper for a full description of the gear. In this pamphlet we are concerned only with the epicyclic portion of the gearbox and with this end in view, Fig. 14 has been prepared. In this figure only the gears, shafts and carrier plates are shown, all bushes, ball-bearings, etc., having been omitted. P is the driving shaft driven from the engine. Q is the driven shaft transmitting the motion to the rear axle. A is the reverse brake drum, revolving freely on the reverse carrier plate M. M, in turn, is keyed to the driven shaft Q and carries the planet pinion B which gears internally with A and externally with the first speed brake drum C. C revolves freely on the first speed carrier plate L. L, in turn, is keyed to the shaft Q and carries the planet pinion D, which gears internally with C and externally with the driving wheel E. E is a double gear wheel keyed to the driving shaft P. F is the second speed planet pinion and is carried by a carrier plate N and an annular wheel J. N has a series of notches on its circumference. These notches fit into the teeth in C, making C and N revolve as one. F gears externally with the driving wheel E and internally with the second speed brake drum G. G, in turn, carries a pin on which the planet pinion H revolves freely. H gears internally with J and externally with the wheel K which is integral with the third speed brake drum. K revolves freely on the driving shaft P. Integral with K there is also the outer portion of a cone clutch, the cone of which slides on the shaft P and is keyed to P. All planet pinions revolve freely on their pins and any of the brake drums A, C, G or K may be fixed by means of band brakes. In the actual box keys are replaced by splined shafts, but keys are assumed in the sketch for the sake of clearness. The numbers of teeth in the various wheels are A—67 teeth, B—19 teeth, C—29 teeth externally and 67 teeth internally, D—22 teeth, E—23 teeth, F—22 teeth, G—67 teeth, H—19 teeth, J—58 teeth and K—20 teeth.

The various gears are obtained as follows :—

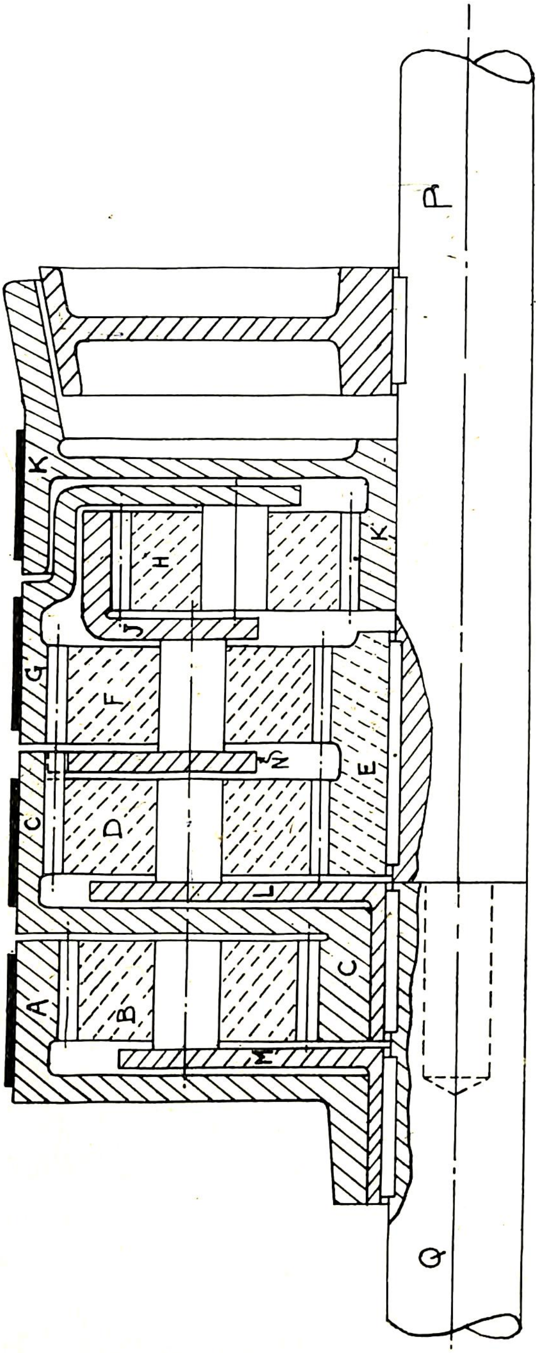


Fig. 14.

Reverse Gear.

The reverse brake drum A is fixed and the power is transmitted from the driving shaft P to the driven shaft Q by the driving wheel E through the wheels ED, C, B, A and the carrier plates L and M. Drawing up the table in the usual fashion we have—

FIXED ↓	REVS. OF					
	A	B	L, M	C	D	E
L, M	$-{}^{29}/_{67}$	$-\ {}^{29}/_{19}$	0	$+\ 1$	$+\ {}^{67}/_{22}$	$-{}^{67}/_{23}$
A	0	$-{}^{29}/_{19}+{}^{29}/_{67}$	${}^{29}/_{67}$	$1\ +{}^{29}/_{67}$	${}^{67}/_{22}+{}^{29}/_{67}$	$-{}^{67}/_{23}+{}^{29}/_{67}$

Note that the carrier plates L and M are in one column together as, since they are both keyed to the driven shaft Q, they will rotate as one.

Velocity ratio of gear

$$= \frac{\text{Revs. of follower LM}}{\text{Revs. of driver E}}$$

$$= \frac{{}^{29}/_{67}}{-{}^{67}/_{23} + {}^{29}/_{67}} = \frac{1}{-({}^{67}/_{23} \times {}^{67}/_{29}) + 1} = \frac{1}{-6{\cdot}73 + 1}$$

$$= -\frac{1}{5{\cdot}73}$$

First Gear.

The first speed brake drum C is fixed and the power is transmitted from the driving shaft P to the driven shaft Q by the driving wheel E through the wheels D and C and the carrier plate L. Drawing up the table in the usual fashion, we have :—

FIXED ↓	REVS. OF			
	E	D	C	L
L	$-\ {}^{22}/_{23}$	$+\ 1$	$+\ {}^{22}/_{67}$	0
C	$-{}^{22}/_{23}-{}^{22}/_{67}$	$1\ -{}^{22}/_{67}$	0	$-\ {}^{22}/_{67}$

$$\text{Velocity ratio of gear} = \frac{\text{Revs. of follower L}}{\text{Revs. of driver E}}$$

$$= \frac{-{}^{22}/_{67}}{-\ {}^{22}/_{23}\ -\ {}^{22}/_{67}}$$

$$= \frac{1}{^{67}/_{23} + 1}$$

$$= \frac{23}{90}$$

$$= \frac{1}{3{\cdot}91}$$

Second Gear.

The second speed brake drum G is fixed and the power is transmitted from the driving shaft P to the driven shaft Q by the driving wheel E through the wheels F, G, D and C and the carrier plate L. Drawing up the table in the usual fashion, we have :—

FIXED	REVS. OF		
↓	E	C. N. J	D
L	$-^{22}/_{23}$	$+^{22}/_{67}$	$+1$
C, N, J	$-^{22}/_{23} - ^{22}/_{67}$	0	$1 - ^{22}/_{67}$
G	$-^{22}/_{23} - ^{22}/_{67} - (^{23}/_{67})\ (^{22}/_{23} + ^{22}/_{67})$		

	REVS. OF		
	L	F	G
L	0		
C, N, J	$-^{22}/_{67}$	$(^{22}/_{23} + ^{22}/_{67})\ (^{23}/_{22})$	$(^{22}/_{23} + ^{22}/_{67})\ (^{23}/_{67})$
G	$-^{22}/_{67} - (^{23}/_{67})\ (^{22}/_{23} + ^{22}/_{67})$		0

The first line of the table is obtained in the normal way by fixing L and giving D one positive revolution. Again N has a series of notches on its circumference which fit into the teeth in C and N and J are connected by the pin carrying the planet pinion F. Hence C, N and J are listed as one in the table. To obtain the second line of the table, CNJ is fixed and its revs $(+^{22}/_{67})$ are subtracted from each item in the first line. In this way we obtain the revs. of E as $(-^{22}/_{23} - ^{22}/_{67})$. But F gears with E and since there are 22 teeth in F and 23 teeth in E, the revs. of F will be *opposite* to those of E and $^{23}/_{22}$ of their value. Hence the revs. of F will be $(-^{22}/_{23} - ^{22}/_{67}) \times -^{23}/_{22} = (^{22}/_{23} + ^{22}/_{67})\ (^{23}/_{22})$. Again F gears with G and the revs. of G will be in the same direction as those of F and $^{22}/_{67}$ of their value. Hence the revs. of G will be

$(^{22}/_{23} + ^{22}/_{67})\ (^{23}/_{22})\ (^{22}/_{67}) = (^{22}/_{23} + ^{22}/_{67})\ (^{23}/_{67})$. The third and last line of the table is obtained by fixing G and subtracting its revs., viz., $(^{22}/_{23} + ^{22}/_{67})\ (^{23}/_{67})$ from each item in the second line. Note that only those values essential to the calculation of the velocity ratio of the train have been entered in the third line.

$$\text{Velocity ratio of gear} = \frac{\text{Revs. of follower L}}{\text{Revs. of driver E}}$$

$$= \frac{-^{22}/_{67} - ^{22}/_{67} - (^{22}/_{67} \times ^{23}/_{67})}{-^{22}/_{23} - ^{22}/_{67} - ^{22}/_{67} - (^{22}/_{67} \times ^{23}/_{67})}$$

$$= \frac{-1 - 1 - ^{23}/_{67}}{-^{67}/_{23} - 1 - 1 - ^{23}/_{67}}$$

$$= \frac{2^{23}/_{67}}{2^{23}/_{67} + ^{67}/_{23}}$$

$$= \frac{2{\cdot}343}{2{\cdot}343 + 2{\cdot}91}$$

$$= \frac{2{\cdot}343}{5{\cdot}253}$$

$$= \frac{1}{2{\cdot}24}$$

Third Gear.

The third speed brake drum K is fixed and the power is transmitted from the driving shaft P to the driven shaft Q by the driving wheel E, through the wheels K, H, J, F, G, D and C, and the carrier plate L. Drawing up the table in the usual fashion we have :—

FIXED ↓	REVS. OF		
	K	H	G
L			
C, N, J			$({}^{22}/_{23}+{}^{22}/_{67})\ ({}^{23}/_{67})$
G	$({}^{22}/_{23}+{}^{22}/_{67})\ ({}^{23}/_{67}\times{}^{58}/_{20})$	$-({}^{22}/_{23}+{}^{22}/_{67})\ ({}^{23}/_{67}\times{}^{58}/_{19})$	0
K	0		

	REVS. OF	
	C, N, J	E
L	$+\ {}^{22}/_{67}$	$-\ {}^{22}/_{23}$
C, N, J	0	$-\ {}^{22}/_{23}-{}^{22}/_{67}$
G	$-\ ({}^{22}/_{23}+{}^{22}/_{67})\ {}^{23}/_{67}$	$-\ {}^{22}/_{23}-{}^{22}/_{67}-({}^{22}/_{23}+{}^{22}/_{67})\ ({}^{23}/_{67})$
K		$-\ {}^{22}/_{23}-{}^{22}/_{67}-({}^{22}/_{23}+{}^{22}/_{67})\ ({}^{23}/_{67})$ $-\ ({}^{22}/_{23}+{}^{22}/_{67})\ ({}^{23}/_{67}\times{}^{58}/_{20})$

	REVS. OF		
	F	D	L
L		$+\ 1$	0
C, N, J	$({}^{22}/_{23}+{}^{22}/_{67})\ ({}^{23}/_{22})$	$1-{}^{22}/_{67}$	$-\ {}^{22}/_{67}$
G			$-\ {}^{22}/_{67}-({}^{22}/_{23}+{}^{22}/_{67})\ ({}^{23}/_{67})$
K			$-\ {}^{22}/_{67}-({}^{22}/_{23}+{}^{22}/_{67})\ ({}^{23}/_{67})$ $-\ ({}^{22}/_{23}+{}^{22}/_{67})\ ({}^{23}/_{67}\times{}^{58}/_{20})$

The first three lines of the table are obtained as for the second gear. In this way the revs. of J are $-({}^{22}/_{23}+{}^{22}/_{67})\ {}^{23}/_{67}$. But H gears with J and the revs. of H will be in the same direction as those of J and ${}^{58}/_{19}$ of their value. Hence the revs. of H will be $-({}^{22}/_{23}+{}^{22}/_{67})\ ({}^{23}/_{67}\times{}^{58}/_{19})$. Again, H gears with K and the revs. of K will be opposite in direction to those of H and ${}^{19}/_{20}$ of their value. Hence the revs. of K will be $+\ ({}^{22}/_{23}+{}^{22}/_{67})\ ({}^{23}/_{67}\times{}^{58}/_{19}\times{}^{19}/_{20})=({}^{22}/_{23}+{}^{22}/_{67})\ ({}^{23}/_{67}\times{}^{58}/_{20})$. The fourth and last line of the table is obtained by fixing K and subtracting its revs., viz., $({}^{22}/_{23}+{}^{22}/_{67})\ ({}^{23}/_{67}\times{}^{58}/_{20})$ from each item in the third line. Note that only those values essential to the calculation of the velocity ratio of the train have been entered in the fourth line.

Then velocity ratio of gear

$$= \frac{\text{Revs. of follower L}}{\text{Revs. of driver E}}$$

$$= \frac{-{}^{22}/_{67} - ({}^{22}/_{23} + {}^{22}/_{67})\ {}^{23}/_{67} - ({}^{22}/_{23} + {}^{22}/_{67})\ ({}^{23}/_{67} \times {}^{58}/_{20})}{-{}^{22}/_{23} - {}^{22}/_{67} - ({}^{22}/_{23} + {}^{22}/_{67})\ {}^{23}/_{67} - ({}^{22}/_{23} + {}^{22}/_{67})\ ({}^{23}/_{67} \times {}^{58}/_{20})}$$

$$= \frac{-{}^{22}/_{67} - {}^{22}/_{67} - ({}^{22}/_{67} \times {}^{23}/_{67}) - ({}^{22}/_{23} + {}^{22}/_{67})\ ({}^{23}/_{67} \times {}^{58}/_{20})}{-{}^{22}/_{23} - {}^{22}/_{67} - {}^{22}/_{67} - ({}^{22}/_{67} \times {}^{23}/_{67}) - ({}^{22}/_{23} + {}^{22}/_{67})\ ({}^{23}/_{67} \times {}^{58}/_{20})}$$

$$= \frac{-{}^{22}/_{23} - {}^{22}/_{23} - {}^{22}/_{67} - ({}^{22}/_{23} + {}^{22}/_{67})\ 2{\cdot}9}{-({}^{22}/_{23} \times {}^{67}/_{23}) - {}^{22}/_{23} - {}^{22}/_{23} - {}^{22}/_{67} - ({}^{22}/_{23} + {}^{22}/_{67})\ 2{\cdot}9}$$

$$= \frac{(4{\cdot}9 \times {}^{22}/_{23}) + (3{\cdot}9 \times {}^{22}/_{67})}{({}^{22}/_{23} \times {}^{67}/_{23}) + (4{\cdot}9 \times {}^{22}/_{23}) + (3{\cdot}9 \times {}^{22}/_{67})}$$

$$= \frac{4{\cdot}68 + 1{\cdot}28}{2{\cdot}79 + 4{\cdot}68 + 1{\cdot}28}$$

$$= \frac{5{\cdot}96}{8{\cdot}75} = \frac{1}{1{\cdot}46}$$

Top Gear.

The cone keyed to the shaft P is engaged with the third speed brake drum K and the whole gear runs "solid," so that the speed of the driven shaft Q is the same as that of the driving shaft P, giving a velocity ratio of 1 : 1.

Neutral Position.

In this position the cone is disengaged and all brake drums are free. The resistance between the tyres and the road and the oil resistance in the transmission act as a brake on the driven shaft Q and prevent it from turning, the driving shaft P simply running idle.

By the courtesy of E.N.V. Engineering Co., Ltd., the author is able to reproduce in Fig. 15 a running gear assembly of this gearbox as fitted on pleasure-car models.

The Differential Gear.

No paper on epicyclic gearing would be complete without a reference to this all-important application. Fig. 16 shows the essential parts of a differential gear. A and C are equal bevel wheels keyed to independent shafts. B and B¹ are bevel planet pinions. D is the "arm" consisting of a box free to turn concentric with the bevel wheels A and C and having a diametral spindle on

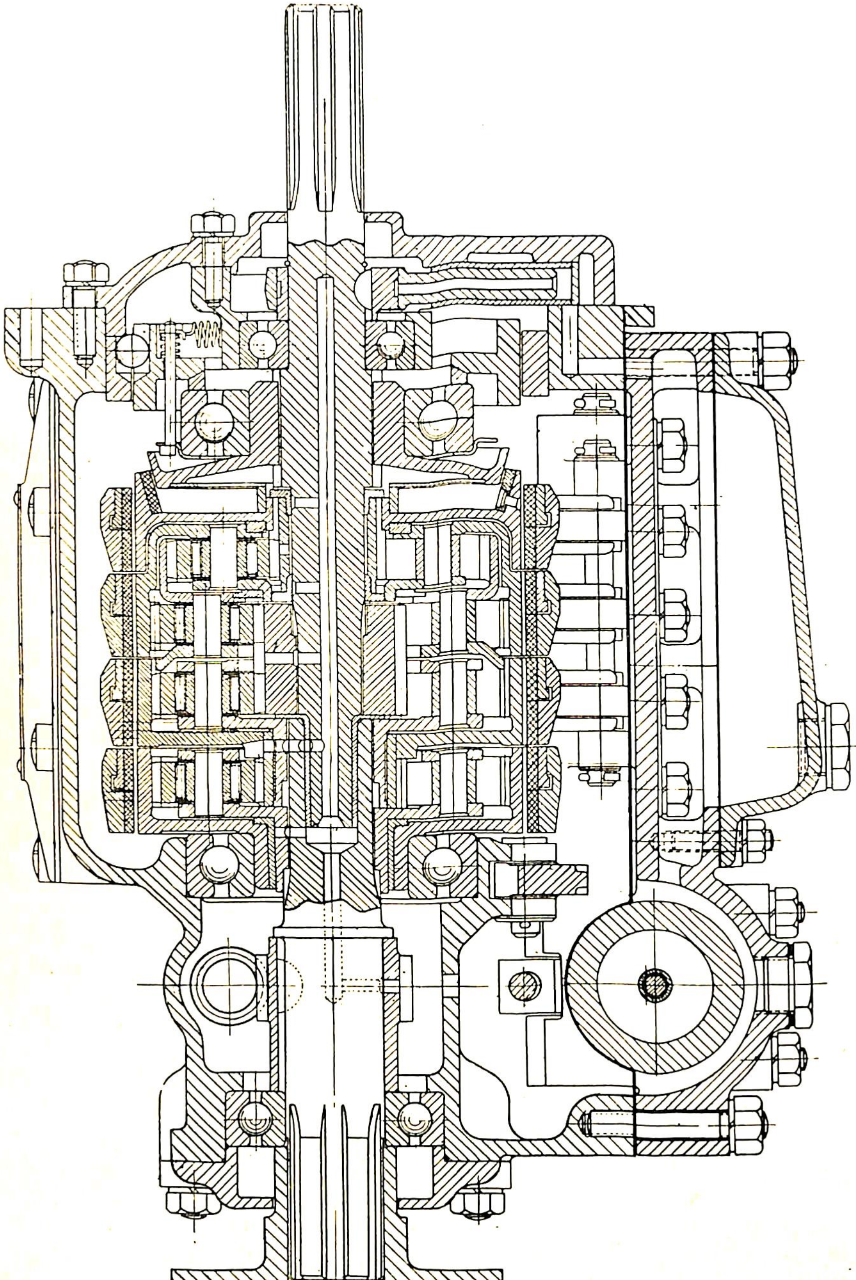


Fig. 15—Running Gear Assembly. By courtesy of E.N.V. Eng. Co. Ltd.

which the bevel pinions B and B^1 turn freely. In drawing up the table for this gear, view the gear in the direction of the arrow and let the letter indicating the wheel denote the number of teeth in it. Again, since A and C are equal, both wheels may be denoted by the letter A and since B and B^1 are equal, both wheels may be denoted by the letter B.

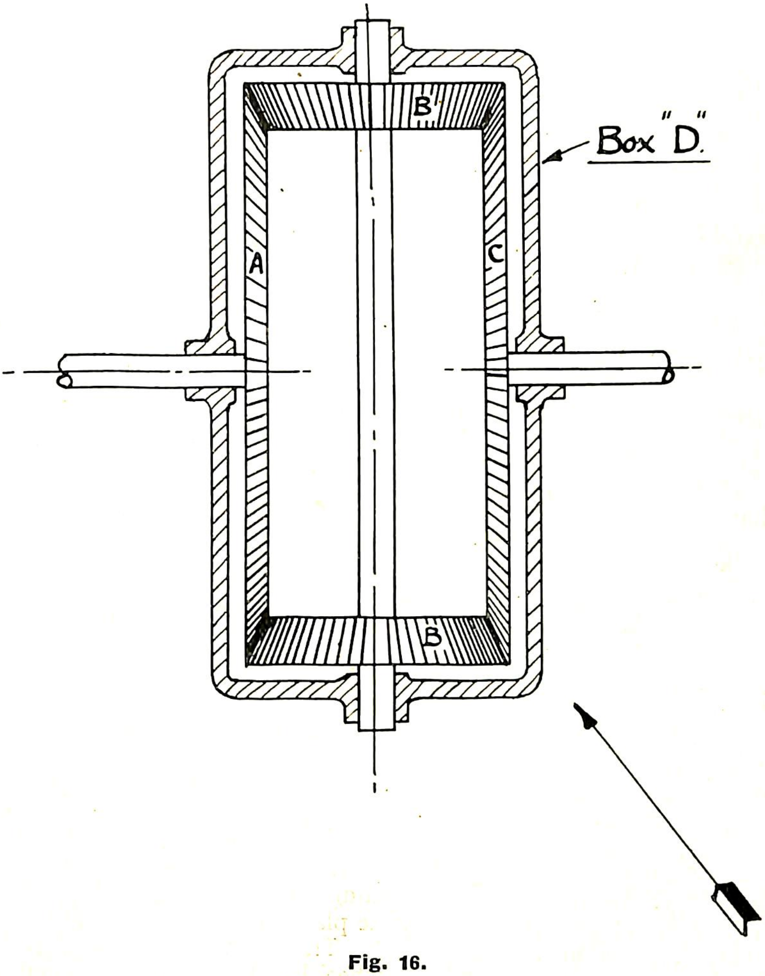


Fig. 16.

Fixed ↓	Revs. of				
	A	B	B^1	C	D
D	$+\frac{B}{A}$	-1	$+1$	$-\frac{B}{A}$	0
B	$\frac{B}{A}+1$	0	$1+1$	$-\frac{B}{A}+1$	$+1$
B^1	$\frac{B}{A}-1$	$-1-1$	0	$-\frac{B}{A}-1$	-1
C	$\frac{B}{A}+\frac{B}{A}$	$-1+\frac{B}{A}$	$1+\frac{B}{A}$	0	$+\frac{B}{A}$
A	0	$-1-\frac{B}{A}$	$1-\frac{B}{A}$	$-\frac{B}{A}-\frac{B}{A}$	$-\frac{B}{A}$
All Free	$\frac{B}{A}+x$	$-1+x$	$1+x$	$-\frac{B}{A}+x$	x

The first line of the table is obtained by fixing the box D and giving the pinion B^1 one positive revolution. The second line is obtained by fixing B and subtracting its revs. (-1) from all the other items in the first line. The third, fourth and fifth lines are obtained in a similar fashion by fixing B^1, C and A in turn. The last line is obtained from the first line by freeing the box D, giving it x revs. and adding x to each of the items in the first line. Note that *in every line* the *revolutions of the box D* are the *mean* of those of the shafts *A and C* and that the direction of rotation of D is that of the faster running shaft.

There are many practical applications of this gear, of which the following are typical :—

(*a*) **As Differential Gear for Motor Car Rear Axle.**

Fig. 17 shows the usual type of rear axle drive for a motor car. In this case the box D takes the form of a bevel wheel E driven by the transmission shaft through the bevel pinion F. E runs freely on the shaft S. The shafts S and T forming the rear axle are connected to the bevel wheels A and C. The bevel wheel E carries two brackets fitted with pins on which the bevel pinions B and B^1 run freely. When the car is running in a straight path the speeds of the shafts S and T are equal, the planet pinions B and B^1 do not revolve and the gear runs "solid." The speeds of the shafts S and T are then the speed of the bevel wheel E. When rounding a curve, however, the speed of the outer shaft must be greater than the speed of the inner if no slip between the wheel and the road is to occur.

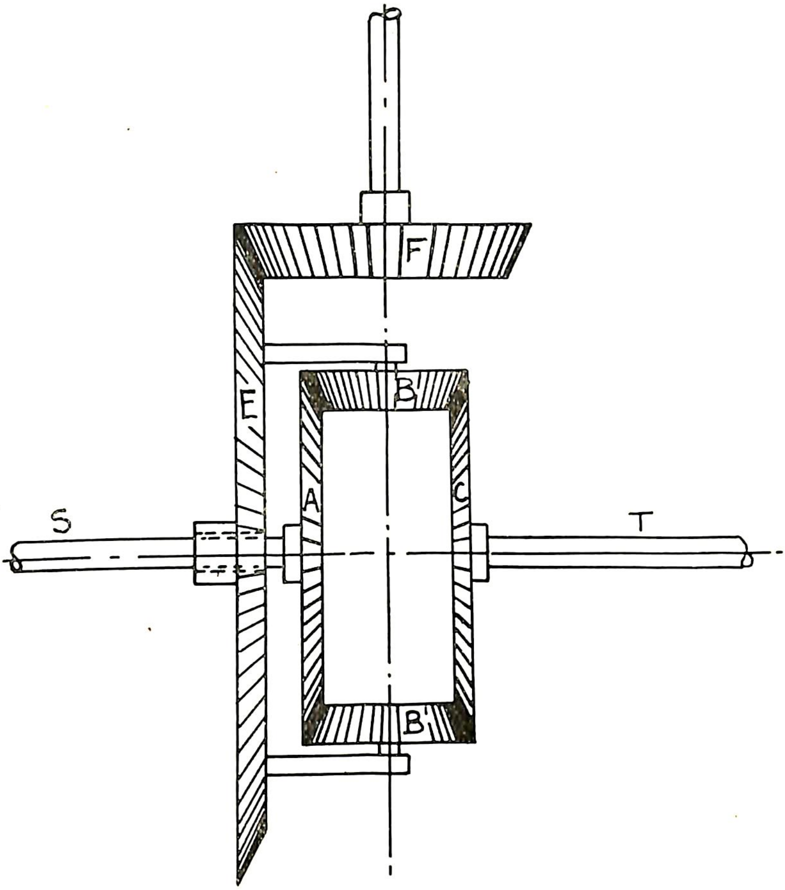


Fig. 17.

The gear permits this by a rotation of the bevel pinions B and B^1. The mean speed of the shafts S and T, however, is still that of the "box" or bevel wheel E.

(*b*) **As Differential Speed Indicator for a Twin-Screw Ship.**

In this case the bevel wheels A and C are driven in opposite directions by the port and starboard engines respectively and the box D carries a pointer moving over a dial. This pointer rotates in the direction of the faster-running engine at one-half of the difference of the speed. For example, if the port engine is running at 104 R.P.M. and the starboard engine at 100 R.P.M., the pointer will move at $\frac{104-100}{2}$ R.P.M., *i.e.*, at 2 R.P.M. in a direction corresponding to the port engine.

(*c*) As a Reversing Gear for Small Engines used on Motor Boats, etc.

In this case the differential gear is used in conjunction with a band brake and cone clutch, as shown in Fig. 18. The box D can be (1) clamped to the driven shaft by means of the cone clutch, or (2) held fixed by a band brake wrapping round the outside of the box. There are three positions of this gear :—

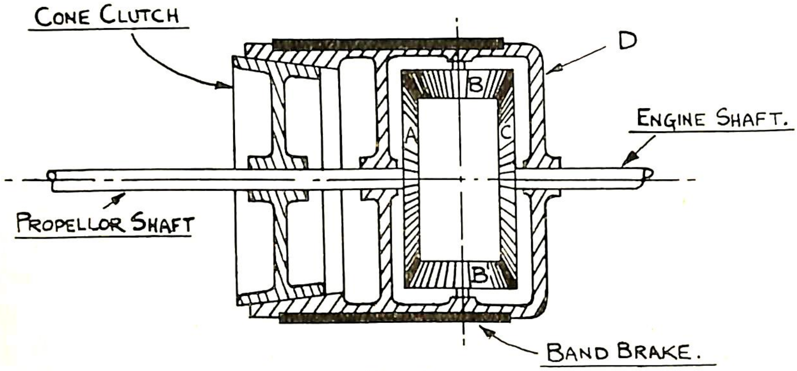


Fig. 18.

(1) *Neutral.*—The clutch and brake are both free. The resistance keeps the propeller shaft from turning and the engine runs idle, driving the box at half speed.

(2) *Ahead.*—Clutch engaged, brake free. The box is now clamped to the propeller shaft and the whole gear runs "solid" at engine speed.

(3) *Astern.*—Clutch free, brake engaged. The box is now fixed and the propeller shaft runs at engine speed *but in the opposite direction.*

Note that in each of the three positions the speed of the box D is the mean of the speeds of the wheels A and C.

Conclusion.

The examples given of the applications of epicyclic gearing in practice will give readers some idea of the relative importance and utility of this type of gearing and it is hoped that a detailed study of this pamphlet will enable engineers to view the subject in its proper perspective and dispel, once and for all, the prevailing idea that an epicyclic job is one to be approached in fear and trembling.

List of A.E.S.D. Printed Pamphlets and Other Publications in Stock.

The Uniflow Engine as a Prime Mover,	Arthur Rice
The Marine Gear Problem from an Under-rated Aspect,	William Sellar
*The Craft of Tracing,	
* (6d to members, others 1/- Post Free).	
Lost Volumes from Cones,	W. R. Thomson
Induction Motor Design,	William Sharp
The Operation of Single-Phase Induction Motors,	G. Windred
*The Design of Ship's Lines and Considerations on Form and Resistance,	R. Allan
* (1/6 to members, 3/- others. Post Free).	
Shearing Machines for Steelworks,	F. H. Roberts
Gear Tooth Forms (reprint),	E. W. Tipple
Graphical Statics,	E. N. Digweed
Hydraulics,	E. N. Digweed
*The Balancing of Engines,	C. C. Pounder
*(2/- to members, 4/- to others. Post Free).	
Vectorial Solution of A.C. Circuits,	A. Garnett
Valve and Reversing Gears,	Beatson Hutchison
Pressure Vessels for High Pressure Boilers (Land Type)	M. B. Hamilton.
*The Internal Combustion Locomotive,	Brian Reed.
* (1/6 to members, 3/- to others. Post Free).	
Insulation and Fittings of Refrigerated Spaces on Shipboard,	R. M. Buchanan
Flywheel Design (for Reciprocating Engines),	Robt. B. Neilson
The Parallel Operation of Diesel-Alternator Sets,	G. H. Bradbury
Introduction to the Theory of D.C. Motors,	G. Windred
Perspective Drawing for Draughtsmen (reprint),	William Sellar
Calculation of Network Short Circuits,	A. Garnett
The Solution of Triangles,	E. N. Digweed
*Deflection of Beams,	E. N. Digweed
* (2/- to members, 4/- to others. Post Free).	
High-Speed Oil-Engine Design,	B. Reed and H. Tanaka
Graphical Methods for Treating Certain Beam Problems (2nd Revised Reprint),	W. R. Needham
*Aeroplane Design,	R. Rodger
* (4/- to members, 6/- to others. Post Free).	
Processes of Seamless Steel Tube Manufacture,	R. S. Betteridge
*Marine Engine Pipe Arrangements,	John M. Baxter
*(1/6 to members; 3/- to others. Post Free).	
Some Notes on Mechanical Design of High-Speed Salient-Pole A.C. Rotors,	W. R. Needham
Some Notes on Deflection,	W. R. Thomson
Engineering Applications of Electron Physics,	G. Windred
The Principle and Design of Precision Gauges for Interchangeability,	R. J. Foster
*Belt Conveyors,	H. Davies
*(2/- to members, 4/- to others. Post Free).	

Steam Turbines,	C. S. Bradshaw
Road Vehicle Performance,	R. W. Collins
The Application of Influence Lines to the Stress Analysis of Beams and Lattice Girders (Part I.),	R. McCrae
The Application of Influence Lines to the Stress Analysis of Beams and Lattice Girders (Part II.),	R. McCrae
*Mechanical Tests for Engineering Materials, *(2/- to members, 4/- to others. Post Free).	A. M. Roberts
*Steam Locomotive Design, *(2/- to members, 4/- to others. Post Free).	D. Patrick.
*Small Craft Types, *(1/6 to members, 3/- to others)	John A. Mavor
*Power Station Electrical Equipment and Layout, *(2/- to members, 4/- to others. Post Free).	T. H. Carr
An Introduction to the Study of Noise Problems,	H. Davies
Design Examples for Young Draughtsmen,	J. W. Hamilton
Positive Rotary Pumps, Compressors and Exhausters,	B. Pugh
The Case-Hardening of Steel,	A. C. Macdonald
Data for the Design of Axial Flow Fans and Pumps,	R. Poole
The Geometry of Conical Pipes, Bends and Joints,	W. Sellar.
*Crystals and the Crystal Structure of Metallic Compounds, *(1/6 to members, 3/- to others. Post Free).	C. G. James
The Safeguarding of Machinery by Design, * (1/6 to members, 3/- to others. Post Free).	E. W. Stott
Epicyclic Gearing,	R. McCrae
*Design of Steel Bridges, *(10/- to members, 15/- to others. Post Free).	F. H. Abrahams
*Compressed Air and Its Machinery, *(4/- to members, 6/- to others. Post Free).	T. H. Plummer

(All not otherwise marked are 1/- to members, 2/- to others, post free).

LIST OF A.E.S.D. DATA SHEETS.

---

1. Safe Load on Machine-Cut Spur Gears.
2. Deflection of Shafts and Beams. } Connected.
3. Deflection of Shafts and Beams (Instruction Sheet) }
4. Steam Radiation Heating Chart.
5. Horse-Power of Leather Belts, etc.
6. Automobile Brakes (Axle Brakes) } Connected.
7. Automobile Brakes (Transmission Brakes) }
8. Capacities of Bucket Elevators.
9. Valley Angle Chart for Hoppers and Chutes.
10. Shafts up to 5¼-in. diameter, subjected to Twisting and Combined Bending and Twisting.
11. Shafts, 5¾ to 26 in. diameter, subjected to Twisting and Combined Bending and Twisting.
12. Ship Derrick Booms.
13. Spiral Springs (Diameter of Rd. or Sq. Wire).
14. Spiral Springs (Compression).
15. Automobile Clutches (Cone Clutches).
16. ,, ,, (Plate Clutches).
17. Coil Friction for Belts, etc.
18. Internal Expanding Brakes. Self-Balancing Brake Shoes (Force Diagram). } Connected.
9. Internal Expanding Brakes. Angular Proportions for Self-Balancing. }
20. Referred Mean Pressure Cuts-Off, etc.
21. Particulars for Balata Belt Drives.
22. ⅞″ Square Duralumin Tubes as Struts.
23. 1″ ,, ,, ,, ,,
24. ¾″ Sq. Steel Tubes as Struts (30 ton yield).
25. ⅞″ ,, ,, ,, (30 ,,).
26. 1″ ,, ,, ,, (30 ,,).
27. ¾″ ,, ,, ,, (40 ,,).
28. ⅞″ ,, ,, ,, (40 ,,).
29. 1″ ,, ,, ,, (40 ,,).
30. Moments of Inertia of Built-up Sections (Tables) } Connected.
31. Moments of Inertia of Built-up Sections (Instructions and Examples). }
32. Reinforced Concrete Slabs (Line Chart) } Connected.
33. Reinforced Concrete Slabs (Instructions and Examples) }
34. Capacity and Speed Chart for Troughed Band Conveyors.
35. Screw Propeller Design (Sheet 1, Diameter Chart) } Connected.
36. ,, ,, ,, (,, 2, Pitch Chart) }
37. ,, ,, ,, (,, 3, Notes and Examples) }
38. Open Coil Conical Springs.
39. Close Coil ,, ,,
40. Trajectory Described by Belt Conveyors.
41. Metric Equivalents.
42. Useful Conversion Factors.
43. Torsion of Non-Circular Shafts.
44. Railway Vehicles on Curves.
45. Chart of R.S. Angle Purlins.
46. Coned Plate Development.
47. Solution of Triangles (Sheet 1, Right Angles).
48. Solution of Triangles (Sheet 2, Oblique Angles).

49. Relation between Length, Linear Movement and Angular Movement of Lever. (Diagram and Notes).
50. ,, ,, ,, ,, ,, ,, (Chart).
51. Helix Angle and Efficiency of Screws and Worms.
52. Approximate Radius of Gyration of Various Sections.
53. Helical Spring Graphs (Round Wire)
54. ,, ,, ,, (Round Wire) } Connected.
55. ,, ,, ,, (Square Wire)
56. Relative Values of Welds to Rivets.
57. Ratio of Length/Depth of Girders for Stiffness.
58. Graphs for Strength of Rectangular Flat Plates of Uniform Thickness.
59. Graphs for Deflections of Rectangular Flat Plates of Uniform Thickness.
60. Moment of Resistance of Reinforced Concrete Beams.
61. Deflection of Leaf Spring.
62. Strength of Leaf Spring.
63. Chart showing relationship of various Hardness Tests.
64. Shaft Horse Power and Proportions of Worm Gears.
65. Ring with Uniform Internal Load (Tangential Strain)
66. Ring with Uniform Internal Load (Tangential Stress) } Connected.
67. Hub Pressed on to Steel Shaft. (Maximum Tangential Stress at Bore of Hub).
68. Hub Pressed on to Steel Shaft. (Radial Gripping Pressure between Hub and Shaft).
69. Rotating Disc (Steel) Tangential Strain
70. ,, ,, ,, ,, Stress } Connected.
71. Ring with Uniform External Load, Tangential Strain
72. ,, ,, ,, ,, Stress } Connected.
73. Viscosity Temperature Chart for Converting Commercial to Absolute Viscosities.
74. Journal Friction on Bearings. } Connected.
75. Ring Oil Bearings.
76. Shearing and Bearing Values for High Tensile Structural Steel Shop Rivets, in accordance with B.S.S. No. 548/1934.
77. Permissible Compressive Stresses for High Tensile Structural Steel, manufactured in accordance with B.S.S. No. 548/1934 } Connected.
78. Velocity of Flow in Pipes for a Given Delivery } Connected.
79. Delivery of Water in Pipes for a Given Head
80. Working Loads in Mild Steel Pillar Shafts.

(Data Sheets are 2d to members, 4d to others, post free).

Orders for Pamphlets, Data Sheets, and Books to be sent to The Editor, *The Draughtsman*, cheques and orders being crossed "Draughtsman Publishing Co., Ltd."

For latest information regarding publications in stock, see Advertisements in *The Draughtsman* and Combined Order Forms, issued annually.